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Semi-Definite Programming for Power Output Control in Wind Energy Conversion System

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Motivation

- Wind Energy has been growing fast in many countries recently.
- Variable speed and variable pitch WECSs give a better performance, since the generator torque and the pitch angle of the turbine blades can be controlled independently and simultaneously.
- Sophisticated control strategy plays an important role in wind energy conversion systems. A well-defined WECS is required.

Model of WECS

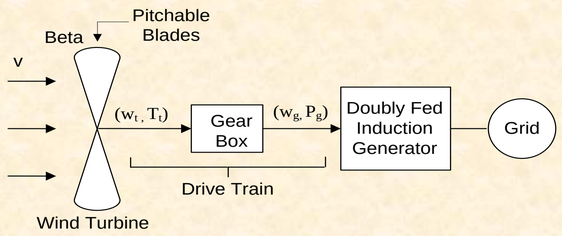


FIG. 1. Wind Energy Conversion System

A. Wind Speed Model

$$v\ t\ = v_m\ t\ + v_t\ t$$

B. Pitch Actuator Model

$$\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta_d$$

C. Generator Model

$$\dot{\Gamma}_g = -\frac{1}{\tau_g}\Gamma_g + \Gamma_g^*$$

D. Aerodynamic System

$$P_t = C_p\ \lambda,\beta\ \frac{\rho\pi R^2}{2}v^3$$

$$\Gamma_t = \frac{P_t}{\omega_t} = \frac{C_p\ \lambda,\beta}{\lambda}\frac{\rho\pi R^3}{2}v^2$$

$$C_p\ \lambda,\beta\ = 0.5176\left(\frac{116}{\lambda_i} - 0.4\beta - 5\right)e^{-21/\lambda_i} + 0.0068\lambda$$

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1}$$

E. Drive Train Model

$$\frac{d\omega_t}{dt} = -\frac{i}{J_t}\Gamma_{tw} + \frac{1}{J_t}\Gamma_t$$

$$\frac{d\Gamma_{tw}}{dt} = k_s i\omega_t - k_s\omega_g - \left(\frac{i^2 B_s}{J_t} + \frac{B_s}{J_g}\right)\Gamma_{tw} + \frac{iB_s}{J_t}\Gamma_t + \frac{B_s}{J_g}\Gamma_g$$

$$\frac{d\omega_g}{dt} = \frac{i}{J_g}\Gamma_{tw} - \frac{1}{J_g}\Gamma_g$$

$$\Gamma_{tw} = k_s\theta_{tw} + B_s\ i\omega_t - \omega_g$$

WECS Linearization

To analyze the system using small signal method, first to linearize the system at a specified operating point.

$$\Gamma_t = f\ \omega_t,\nu,\beta\ = f\ \omega_{t,op},\nu_{op},\beta_{op}$$

$$\Delta\Gamma_t = \frac{\partial f}{\partial\omega_t}|_{op}\ \Delta\omega_t + \frac{\partial f}{\partial\nu}|_{op}\ \Delta\nu + \frac{\partial f}{\partial\beta}|_{op}\ \Delta\beta + \left[\frac{\partial f}{\partial\omega_t}|_{op}\ \Delta\omega_t + \frac{\partial f}{\partial\nu}|_{op}\ \Delta\nu + \frac{\partial f}{\partial\nu}|_{op}\ \Delta\beta\right] + \cdots = L_\omega\Delta\omega_t + L_\nu\Delta\nu + L_\beta\Delta\beta$$

$$\begin{cases} x = \left[\Delta\omega_t\ \Delta\omega_g\ \Delta\Gamma_{tw}\ \Delta\Gamma_g\ \Delta\beta\right] \\ u = \left[\Delta\Gamma_g^*\ \Delta\beta_d\right] \\ y = \left[\Delta\omega_g\ \Delta P_g\right] \end{cases} \Rightarrow \begin{cases} \dot{x}\ t\ = \tilde{A}x\ t\ + \tilde{B}_u u\ t\ + \tilde{B}_\nu\Delta\nu\ t \\ y\ t\ = \tilde{C}x\ t \end{cases}$$

$$\begin{cases} x\ k+1\ = A_d x\ k\ + B_u u\ k\ + B_d d\ k \\ y\ k\ = C_d x\ k \end{cases}$$

Semi-Definite Programming

$$\min V_N\ x_0,\mathbf{u},\mathbf{d}\ = \frac{1}{2}\left[\sum_{k=1}^{N} q_1\ \omega_{g,rat} - \omega_g\ _k^2 + q_2\ P_{g,rat} - P_g\ _k^2 + \sum_{k=0}^{N-1} r_1\ \Delta T_g^*\ _k^2 + r_2\ \Delta\beta_d\ _k^2\right]$$

$$V_N\ x(0),\mathbf{u},\mathbf{d}\ = \left[\sum_{k=1}^{N} x(k)^T Qx(k) + \sum_{k=0}^{N-1} u(k) - u(k-1)\ ^T R\ u(k) - u(k-1)\right]$$

$$\begin{cases} \beta_{\min} \le \beta_d\ k\ \le \beta_{\max} \\ \Delta\beta_{\min} \le \Delta\beta_d\ k\ \le \Delta\beta_{\max} \\ P_{g,\min} \le P_g\ k\ \le P_{g,\max} \\ \omega_{g,\min} \le \omega_g\ k\ \le \omega_{g,\max} \end{cases}$$

Problem 1: Assume wind disturbance is Gaussian

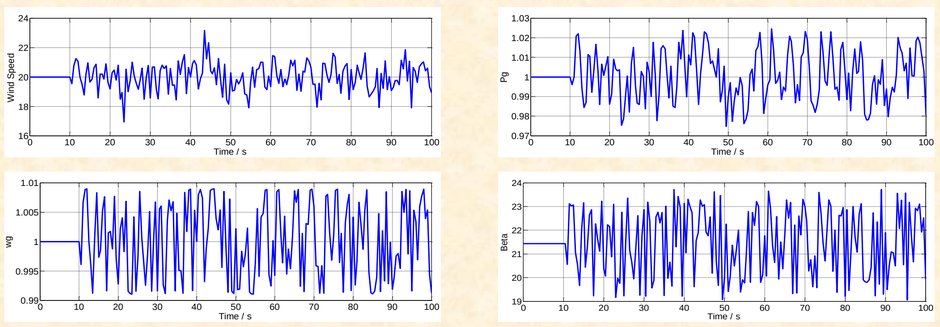
$$\mathbf{u}(x_0) := arg\ min_{\mathbf{u}}\ \mathrm{E}_d V_N$$

Problem 1: Assume wind disturbance is Norm Bounded

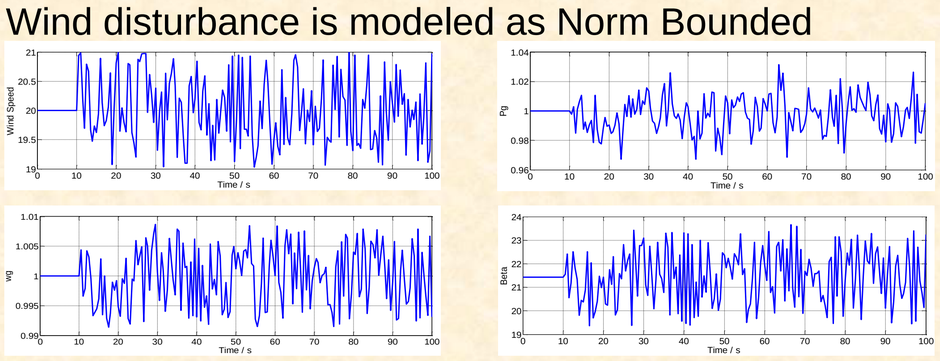
$$\mathbf{u}(x_0) := arg\ min_{\mathbf{u}}\ \max_{d\in\mathrm{D}_\gamma} V_N$$

Simulations

Wind disturbance is modeled as Gaussian



Wind disturbance is modeled as Norm Bounded



Conclusions

- A new control strategy based on SDP is proposed for WECS power control in the full load region. The SDP method solves a stochastic problem by minimizing the expectation of the cost function using the statistics of Gaussian disturbance.
- When the disturbance is modeled as Gaussian distribution, SDP gives comparable results.
- When the wind speed error is modeled as norm bounded without a known distribution, this likely represents a more realistic assumption in practice. The results are also promising.

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