Introduction
Our flywheel sizing studies for grid storage highlight the well-recognized benefit that accrues when the losses are reduced in flywheels. To help provide the engineering knowledge to design robust systems, this project presents a reduced order model for a permanent magnet (PM) and high temperature superconductor (HTSC) in an axisymmetric frame to determine static and dynamic force response. The model is formulated as a bond-graph to be used for system models, where the nonlinear force-displacement interactions are important for stability analysis and control design. The development of the reduced order model is based on the mechanical and electromagnetic interaction between a permanent magnet and bulk HTSC. Performance of the proposed reduced order model is compare to FEM analysis and experimental tests to confirm the static and transient performance.

Model Development
The HTSC puck is modeled as discrete, nested, superconducting rings shorted on themselves. The permanent magnet is modeled by discrete current loops to represent the equivalent surface currents and the resultant magnetic fields in free space. This model assumes that the permanent magnet puck is concentric to the bulk HTSC, which allows the use of an axisymmetric model. Based on conventional formulations [1], the magnetic potential due to a current loop can be calculated at any axial and radial location with respect to loop in free space. From this calculation of magnetic potential, the magnetic flux, and change in flux with respect to axial location of the permanent magnet can be calculated for each discrete superconducting ring of the bulk puck.

Reduced Order Dynamic Model and Bond Graph Representation
Bond graphs are highly useful for modeling systems across multiple energy domains, such as the mechanical and magnetic coupling of the PM-HTSC system [2]. The far left 1-junction of the bond graph below represents the mechanical dynamics of the levitated PM mass. The C gyrator elements represent the electromechanical coupling forces which levitate the PM mass and induce currents within the nested HTSC rings. Each discrete ring of the bulk HTSC is represented by a 1-junction and connected to the other rings via mutual inducance matrix, represented by the large / element. A nonlinear resistance term is also tied to each HTSC ring element to describe the rapid rise in resistivity once the critical current density, Jc is exceeded [3].

Comparison to FEA
To verify performance of the reduced order model, comparisons were made to the finite element method (FEM). The FEM algorithm [4] which utilizes the critical state model, was used for this analysis. The system in this study consisted of a cylindrical PM moving through a bulk HTSC ring at a constant velocity. The hysteretic nature of the force-displacement profile is captured by both analysis techniques.

Experimental Validation
An experiment was performed to validate the model performance by a drop test. This procedure creates a dynamic response by means of a step input, where the weight of the PM is quickly transferred to the magnetic interaction between the PM and HTSC. Tests were performed at different load weights and initial heights above the bulk HTSC. Time domain and frequency response predictions from the model matched experimental results.

The Department of Mechanical Engineering at The University of Texas at Austin
1 The Center for Electromechanics at The University of Texas at Austin
2 The Department of Mechanical Engineering at The University of Texas at Austin

References