Subsurface Monitoring of Geological CO\textsubscript{2} Storage

Investigators
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Abstract
Geological sequestration can be used to reduce greenhouse CO\textsubscript{2} emissions to the atmosphere. The three potential geological storage options are depleted oil and gas reservoirs, deep saline aquifers, and unminable coal beds. Subsurface monitoring will be necessary for leak detection, to provide information for optimizing injection, and to support the safety case that will be required for site licensing. Leak detection is probably the main concern, though other applications such as monitoring for verification of sequestered volumes may be important. Although our feasibility study show that subsurface CO\textsubscript{2} monitoring may be possible with a number of different geophysical methods, we believe that seismic is the key technology because of its effectiveness at detecting small volumes, its pervasive applicability in different geological settings, and its potential for high spatial- and temporal-resolution as may be needed during the lifetime of an storage site.

The focus of our research is the development of a novel approach to continuous 4-D seismic monitoring. In order to make continuous observations practical, we propose to use sparse spatial coverage from low-power coded or continuous-wave sources. Our research indicates that the quality of sparse data imaging can be improved through use of special imaging algorithms that take advantage of temporal regularization and predictive model evolution. We have developed simulation tools and used them to simulate our monitoring strategies with high-performance computer clusters. The next activity is to test this monitoring approach on field data sets.

In this report, we present the results from the feasibility analysis that lead to our focus on seismic methods. Then we present a coupled approach for the study and development of the strategy for continuous-in-time seismic monitoring. This coupled approach includes the development of a new laboratory method for low frequency acoustic rock properties, especially attenuation, and the development of multi-scale simulation tools for seismic wave propagation in visco-elastic media. Finally, we present our new and novel approach for continuous 4-D seismic monitoring that includes arrays of embedded sources and detectors and strategies for adaptive imaging. Conclusions: (1) Sparse spatial coverage is required to enable continuous temporal coverage; (2) Improved temporal coverage may compensate somewhat for sparse spatial coverage; (3) Circular arrays provide the best spatial coverage; (4) Frequency domain data acquisition and imaging methods improve 3-D coverage from sparse datasets.
1. Introduction

CO₂ Sequestration is the process of capturing, separating, transporting, and storing waste CO₂. The motivation is that CO₂ storage mitigates its contribution to global climate change. Three principal types of geological storage formations are being considered, namely depleted oil and gas reservoirs, deep saline and cold beds. Appropriate levels of monitoring of storage sites will be needed in order to help manage the injection process and to insure public safety in the event of an inadvertent leak like that illustrated in Figure 1.1.

**Figure 1.1:** This drawing of a CO₂ storage site illustrates the need subsurface monitoring. The layers above the reservoir provides sealing and flow barriers. A fault indicates a possible CO₂ leak path.

Geological sequestration tests have been performed worldwide. Commercial scale projects include Sleipner, Norway, Weyburn, Canada, In Salah, Algeria, and Snohvit, Norway. At Sleipner, CO₂ is captured from an off-shore natural gas processing platform and injected into a saline formation. Five 3-D seismic surveys were acquired in 1994 (baseline), 1999, 2001, 2002, and 2005 for monitoring the subsurface storage. Seismic images have clearly shown the CO₂ plume and verified containment. Weyburn is an enhanced oil recovery (EOR) project. The CO₂ used at Weyburn is captured in North Dakota and piped across the US-Canada border to the Weyburn oilfield in Saskatchewan. Various monitoring activities are conducted with this EOR project. At In Salah, CO₂ is captured from natural gas processing and re-injected to enhance natural gas recovery. It is a test-bed for monitoring technologies. Examples of pilot scale sequestration tests are K12B (Netherlands, gas reservoir, started in 2004), Otway (Australia, gas reservoir, 2006), RECOPOL (Poland, coal seem, 2004), Hokkaido (Japan, enhanced coal bed methane, 2004), and CASTOR (Europe, capture and storage, 2004).

4-D seismic imaging plays a key role in current subsurface monitoring strategies for CO₂ geological storage. In these projects, complete 3-D seismic surveys are repeatedly acquired, typically at time intervals of several years, to detect the subsurface changes
caused by the injection of CO2. There are two problems with this typical approach with regards to routine monitoring: the cost of mobilizing and implementing repeated 3-D surveys is expensive, especially for routine waste monitoring that is expected to run for decades; and repeated surveys at intervals of years or more will not be effective for safety purposes, especially leak detection, and is possibly unlikely to meet the expected terms of a site licensing agreement.

The objective of our project is to develop new and innovative strategies that are well suitable to continuous seismic monitoring of CO2 storage. We say our goal is to develop a true 4-D monitoring strategy, where time is the sampled 4th dimension. We first investigated a variety of geophysical monitoring methods including seismic, electromagnetic, gravity, and deformation, which are techniques that have potential for use in geological CO2 sequestration (Appendix A). We found that each of these methods have some potential for the monitoring, but concluded that seismic methods have the widest applicability in different geological environments and provide the highest spatial resolution for leak detection. Therefore we selected seismic methods for further investigation in this project. Though technically capable of monitoring CO2, the routine application of conventional seismic methods faces the challenge of routine weekly or daily monitoring. A new paradigm for seismic monitoring is required.

Once seismic was chosen, our project then focused attention on a simulation study of imaging strategy that could be implemented as a true 4-D monitoring scenario, in this case, primarily for depleted oil and gas fields or saline aquifers. We use the best knowledge for the changes in bulk rock properties caused by the injection of CO2. The flow of our project activities is illustrated in Figure 1.2.

**Figure 1.2:** The development of seismic monitoring strategies involves three tasks: Rock properties analysis, survey simulation, and imaging. To test the approach, one could use time-dependent flow simulation results or surrogate models.

In seismic monitoring, we detect changes in acoustic impedance caused by changes in CO2 saturation in the formation. Injected CO2 changes wave speed and density of water-saturated geological formations. Attenuation may change also but much less is know
about it and laboratory measurements are required before we can estimate the magnitude of the changes. Seismic monitoring may detect these changes and then infer information about saturation or storage conditions. Leak detection presents a major conflict in that to be effective, monitoring must be able to detect small temporal changes at small spatial scales in a volume that grows with time. Seismic methods address this conflict by providing a variety of geometries, frequencies, and resolutions that are complementary and useful. These methods include surface-based reflection seismic, borehole vertical seismic profiles and cross-well seismic, passive micro-seismic, and sonic logs. Each represents a tradeoff between resolution and the volume of the subsurface sampled.

In our opinion, the monitoring method must incorporate and utilize predicted behaviors say from flow simulators but must adapt to changing reservoir conditions say for leak detection. In effect, these needs are met by combining reconnaissance surveys with rapidly deployable resources for high-resolution surveys when problems are detected or suspected. To that end, we propose embedding the sources and detectors in and about the storage area, surface- and borehole-based seismic instrumentation. The reconnaissance surveys, i.e., sparse spatial coverage, would be implemented daily to produce high temporal resolution but low-resolution spatial resolution for leak detection. If unexpected changes are detected in regions of the storage site, existing sources and detectors already embedded can be rapidly activated to produce a high spatial resolution of the targeted region. Circular and cross arrays are proposed as examples of sparse observation systems. We must take the advantage of the temporal evolution of the subsurface model and apply temporal regularization and other algorithms to improve the imaging quality of the spatial sparse data. Further, through embedding we can employ low-power sources that radiate either continuous wave signals or coded waveforms. Before applying the proposed concepts to the field tests, we have performed numerous simulation studies. These simulation results are presented in this report.

Finally, we note that the applicability of seismic methods for CO2 monitoring at a specific site depends on the degree or magnitude of change in seismic rock properties, e.g., velocity, density, and attenuation. For this reason, an understanding of seismic rock properties under the condition of CO2 saturation is of great importance to monitoring. Using well-established rock property models, we have performed sensitivity analyses to evaluate the feasibility of monitoring of CO2 storage. We also initiated preliminary development of a pioneering laboratory method, differential acoustical resonance spectroscopy or DARS, for measuring the acoustic properties of small samples of rocks at low frequencies. The current DARS implementation operates at about 1000 Hz on 1 inch cylindrical plugs.
2. Seismic Rock Properties

Let us first investigate the seismic rock properties associated with CO₂ saturation using well-established rock physics models. The models we use to describe the seismic properties of formation rocks undergoing CO₂ injection are fluid substitution from Gassmann [3] with stress-dependence from Eberhart-Phillips [4]. In seismic monitoring the changes we may detect are changes in velocity, reflectivity, and possibly attenuation. In general, the changes in seismic properties are a result of saturation changes and changes in the effective stress. This study helps us understand if the seismic CO₂ monitoring is feasible. Details from the feasibility study are given in Appendix A for seismic and the other geophysical monitoring methods.

2.1 Seismic Model for Brine Aquifers and Depleted Oil and Gas Fields

The seismic properties of the pore fluids that we are concerned with are density and the bulk modulus. The properties of the brine and oil initially present in the formation are fairly insensitive to reservoir conditions while the seismic properties of CO₂ are much stronger functions of pressure and temperature (Figure 2.1). We use the relations collected by Batzle and Wang [2] to estimate the seismic properties of the fluids, e.g., oil, brine, and hydrocarbon gas. Gassmann’s fluid substitution is a low frequency theory, which allows one to determine the effect of pore fluid changes on rock moduli. Using the above effective fluid properties in Gassmann’s equation [3] along with the mineral modulus and the dry rock modulus, one can solve for the saturated moduli with

\[
\frac{K_{\text{sat}}}{K_0 - K_{\text{sat}}} = \frac{K_{\text{dry}}}{K_0 - K_{\text{dry}}} + \frac{K_\beta}{\phi(K_0 - K_\beta)} \quad \text{and} \quad \mu_{\text{dry}} = \mu_{\text{sat}},
\]

where \(K_0, K_{\text{dry}},\) and \(K_{\text{sat}}\) are the mineral, dry rock, and saturated bulk moduli, respectively. \(\phi\) is the porosity and \(\mu\) is the shear modulus, which is unchanged upon fluid substitution under Gassmann’s theory. The saturated density also changes as a result of changing the pore fluid, and can also be calculated from

\[
\rho_{\text{sat}} = \phi \rho_\beta + (1 - \phi) \rho_{\text{dry}}
\]

To model the stress-dependence of fractured rocks we use the results of Eberhart-Phillips [4]. Their work is based on data gathered by Han [5] on the stress-dependent velocities of 64 sandstone samples. In practice, stress-dependence will need to be determined as part of site characterization. Eberhart-Phillips used only sandstone data, but a similar stress-dependence may occur in fractured carbonates. They found the following empirical relation for compressional and shear velocity as a function of porosity, clay content, and effective pressure:

\[
V_p = 5.77 - 6.94\phi - 1.73\sqrt{C} + 0.446\left(P_e - e^{-16.7P_e}\right)
\]
In these two expressions, \( C \) is the mineral fraction of clay, \( P_e \) is the effective pressure in kbar and \( V_p \) and \( V_s \) are in km/s. Figure 2.2 illustrates the effect of changing effective stress on a particular sample, StPeter1. These data were collected for water-saturated rocks.

\[
V_s = 3.70 - 4.94\phi - 1.57\sqrt{C} + 0.361\left(P_e - e^{-16.7P_e}\right) \tag{2.3b}
\]

Combining Gassmann and Eberhart-Phillips allows one to predict the changes from increasing pore pressure and changing saturation with injection and compare the two effects. Figure 2.3 and Figure 2.4 display the results of numerical experiments on a stress-dependent sandstone and a stiffer, unfractured carbonate undergoing CO2 flooding. The top curve in each plot is Gassmann, while each of the other curves assumes a linear increase in pressure with CO2 saturation. Each curve begins at the same reference pore pressure and at zero CO2 saturation. The first thing to notice is that the stiffer rock has a much smaller percent change in velocity, meaning that any changes will be much harder to detect. Also important is that in the fractured sandstone approximately half of the compressional velocity change results from saturating changes and half from pressure effects, while the shear velocity is more affected by pressure changes, which agrees with published results [6].

Figure 2.1: Compressional velocities in (a) brine and oil, and (b) CO2 as a function of pressure and temperature.

Figure 2.2: Data from Eberhart-Phillips for the StPeter1 sample.
**Figure 2.3:** Calculated (a) compressional and (b) shear velocities with CO2 saturation using Gassmann fluid substitution and sandstone with stress dependence.

**Figure 2.4:** Calculated (a) compressional and (b) shear velocities with CO2 flooding using Gassmann fluid substitution for a stiff unfractured carbonate rock with stress dependence.
2.2 Lab Data

Wang and Nur [7] conducted laboratory experiments on sandstone samples under hydrocarbon saturated and CO₂ flooded saturations. The samples were initially saturated with n-hexadecane then flooded with CO₂ leaving approximately 30% residual oil. The confining stress was kept constant at 20 MPa while the pore pressure was increased from approximately 0 to 18 MPa. The results for the Beaver No.7 sample are shown in Figure 2.5a. Figure 2.5b shows the simulated results from our model.

The compressional velocities display similar qualitative behavior while the shear velocities exhibit some striking differences. From Gassmann theory we predict that the shear modulus is unchanged upon flooding, and any velocity change will be the result of density changes. As less dense CO₂ is displacing hydrocarbon oil we expect that flooding will always increase shear velocity. The unexpected behavior of the shear velocity curves in the lab data can be attributed to high frequency viscous effects; Gassmann is a zero frequency equation and cannot always describe sample behavior at laboratory frequencies. Measurements made at field frequencies are expected to show more Gassmann like behavior.

![Figure 2.5: A comparison between (a) lab data from Wang [5], Beaver No. 7 and (b) our stress-dependent fluid substitution model. Black lines are isotherms for hydrocarbon saturated rocks and blue lines are isotherms for CO₂ flooded rocks. Confining pressure for all plots is 20 MPa.](image)

2.3 Reservoir Scale Simulations

Figure 2.6 shows impedance and reflectivity images that are created according to our simple injection model (Figure A-1 in Appendix A). Using the reflectivity time-series and a source wavelet in a convolution model we can create synthetic seismic reflection images (Figure 2.7). In these images we can clearly see the reflector pull-down (Figure 2.7b) from the lower velocities in the CO₂ saturated region and the bright spot associated with the presence of CO₂ (Figure 2.7c).

Three of the principal seismic methods being considered for monitoring sequestration are reflection seismic, velocity tomography, and microseismic. Reflection seismic and crosswell tomography are both expensive, high-resolution techniques. Crosswell seismic imaging techniques have been employed before to monitor CO₂ injection in EOR at the
McElroy Field in West Texas. The seismic survey and the accompanying rock physics study showed that a several percent change was both present and detectable [6, 8, 9] using tomographic techniques.

Figure 2.6: This compressional wave impedance model (a) of the reservoir has bounding shale layers. In this case the only significant and detectable contrasts appear at the sand shale interface because of the smooth velocity variations inside the reservoir. (b) “Reflectivity series” in time for the impedance model after 10 years does show a velocity pull down due to the CO2.

Figure 2.7: (a) baseline, (b) repeat, and (c) differential synthetic seismic images produced using a convolution model applied to the CO2 impedance model.

Micro-seismic monitoring involves using fixed geophones to continuously monitor a formation, providing a real-time image of CO2 movement. This is a relatively inexpensive passive technique, which detects elastic waves resulting from fracture formation or reopening associated with the injection. Fracture formation is, in turn, strongly dependent on pressure changes and rock type, so it may only be useful in low permeability, low porosity rocks where significant pressure changes are expected to occur [10].

From the rock physics modeling discussed previously (Figure 2.3), the bulk of the velocity changes resulting from saturation effects occur with only a small amount of CO2 in the pore space. This means that differentiating 20% saturation from 60% saturation will be much more difficult than detecting the presence of CO2. For this reason the principal usefulness of seismic monitoring will be in leak detection and for monitoring CO2 migration rather than mass balance. Seismic should be able to detect thin layers of CO2 [11], meaning that migration paths should show up clearly in a reflection survey and that presence of CO2 in overlying aquifers should be easily detectable.
Both pressure and saturation effects will be more noticeable in softer rocks, but nonetheless seismic is still the most viable technique for most settings. Changes in seismic properties are not very dependent on initial pore fluid so there is little difference between its use in aquifers and depleted oil fields. The presence of hydrocarbon gas in the pore space, however, may render seismic monitoring useless. The large initial drop in velocity with increasing CO₂ saturation is a result of the high compressibility of the CO₂ making the effective fluid have more gas-like compressibility. If hydrocarbon gas is also present, the effective fluid already has gas-like compressibility and the addition of CO₂ may not have any noticeable effect.

2.4 Rock Properties from Differential Acoustic Resonance Spectroscopy

Laboratory measurement on rock properties helps to interpret field data. However, the existing laboratory studies [6, 7, 38] on effects of CO₂ saturation are quite limited. Another problem with existing lab measurements, including those discussed above, is the gap between the lab frequency (~500 kHz) and the field frequency (~50 Hz). When we apply the laboratory estimates to field measurements of seismic velocity or attenuation to CO₂ saturation, this frequency gap may lead to problems.

The objective of this task is to measure the acoustic properties of small samples of rock in the laboratory at frequencies nearer to the field seismic. We use a unique laboratory method, called Differential Acoustic Resonance Spectroscopy or DARS [39, 40], which makes low frequency measurements on rock properties for a small sample. DARS provides estimates of sample compressibility (κ), inertial density (ρ), and quality factor (Q). These parameters can be interpreted in terms of changes in the fluid and flow properties of a porous medium. Moreover, DARS is ideally suited for irregularly shaped samples such as cuttings or odd shaped pieces of coal. Despite these advantages, DARS is a relatively new method and must be further developed if it is to be used for these purposes.

DARS exploits the perturbations in the resonance frequencies of a fluid-filled cavity to estimate the acoustical properties of a sample loaded in the cavity. In our proof-of-concept system, we use a cylindrical cavity of the type illustrated in Figure 2.8. This system has a resonant frequency around 1100 Hz and can be used with samples as small as 1 cc. In the following, we give a brief explanation of the DARS concept and then show some results.

Assume that a fluid-filled cavity has an angular resonant frequency ω₀. When a sample is loaded into the cavity, the resonant frequency is changed to be ωₛ. Figure 2.9 shows two DARS resonance spectra that correspond to cases, i.e., with and without sample. The relationship between the frequency change and the sample properties can be described, to first order, by the perturbation equation [41]:

\[
ω_s^2 - ω_0^2 = -ω_0^2 \left( \frac{V}{V_c} \right) \frac{\Gamma}{\Lambda} \delta κ - ω_0^2 \left( \frac{V}{V_c} \right) \frac{\Gamma}{\Lambda} \delta ρ .
\] (2.4)
where $\delta \rho = (\rho_s - \rho_0)/\rho_s$; $\delta \kappa = (\kappa_s - \kappa_0)/\kappa_0$; \( \langle f \rangle = \iiint_{V_s} f(x, y, z) dV / V_s \); \( p \) and \( u \) are the corresponding acoustic pressure and particle velocity of the fluid inside the cavity; \( \Gamma \) and \( \Lambda \) are constants for a given cavity; \( V_s \) is the sample volume, and \( V_c \) is the cavity volume. Here \( \rho_0 \) and \( \rho_s \) are fluid and sample densities; \( \kappa_0 \) is the fluid compressibility defined by \( \kappa_0 = 1/\left( \rho_0 v_0^2 \right) \); and \( \kappa_s \) is the sample compressibility given by \( \kappa_s = 1/\left[ \rho_s (v_p^2 - 4 v_s^2 / 3) \right] \), where \( v_p \) and \( v_s \) are the P- and S-wave velocities of the sample. The frequency change is different for different sample locations in the cavity; therefore, multiple measurements in different locations can be used for finding both \( \delta \rho \) and \( \delta \kappa \). Unknown constants \( \langle p \rangle^2 / \Gamma \) and \( \langle u \rangle^2 / \Lambda \) in equation 1.4 can be determined with known samples through a calibration procedure. The quality factor of the cavity can be obtained from

\[
Q_c = f / W, \tag{2.5}
\]

where \( f \) is the resonance frequency and \( W \) is the linewidth of the resonance curve. The sample quality factor \( Q_s \) can be derived from \( Q_c \) and equation 1.4 by introducing complex frequency, complex compressibility, and complex density \( [39] \).

We used equation (1.4) to estimate the compressibility for five solid or impermeable samples and eight permeable (or porous) samples. The results for impermeable samples, listed Table 1.1, show that the DARS estimation agrees well with the ultrasonic (~1 MHz) measurements as expected. However, the results for the permeable samples, listed in Table 1.2, shows that the DARS estimation differs with the ultrasonic estimate, and the difference between the two measurements seems depending on the sample permeability. This observation lead us to a new method for permeability measurement.

Figure 2.8: Diagram of DARS setup. It includes computer-controlled sample positioning and swept frequency data acquisition. The cylindrical cavity with two ends open is in a tank filled with fluid.
Figure 2.9: DARS resonance spectra. The perturbation of a small sample causes both the resonant frequency and peak width changed.

Table 1.1 Acoustical properties of five solid materials.

<table>
<thead>
<tr>
<th></th>
<th>(\rho) (kg/m(^3))</th>
<th>(v_p) (m/s)</th>
<th>(v_s) (m/s)</th>
<th>(\kappa_{\text{ultrasound}}) (GPa(^{-1}))</th>
<th>(\kappa_{\text{DARS}}) (GPa(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2700</td>
<td>6400</td>
<td>3093</td>
<td>0.01334</td>
<td>0.01351</td>
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<tr>
<td>Teflon</td>
<td>2140</td>
<td>1404</td>
<td>750</td>
<td>0.3831</td>
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<td>Delrin</td>
<td>1420</td>
<td>2360</td>
<td>1120</td>
<td>0.1808</td>
<td>0.1838</td>
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<td>PVC</td>
<td>1380</td>
<td>2293</td>
<td>1230</td>
<td>0.2237</td>
<td>0.2257</td>
</tr>
<tr>
<td>Lucite</td>
<td>1180</td>
<td>2692</td>
<td>1550</td>
<td>0.2096</td>
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</tr>
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</table>

Table 1.2 Acoustical properties of eight porous rock samples

<table>
<thead>
<tr>
<th></th>
<th>(\rho) (kg/m(^3))</th>
<th>(phi) (%)</th>
<th>(K) (mD)</th>
<th>(v_p) (m/sec)</th>
<th>(v_s) (m/sec)</th>
<th>(\kappa_{\text{ultrasound}}) (GPa(^{-1}))</th>
<th>(\kappa_{\text{DARS}}) (GPa(^{-1}))</th>
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<tr>
<td>Berea 15</td>
<td>2287</td>
<td>20.85</td>
<td>370</td>
<td>3530</td>
<td>2008</td>
<td>0.06172</td>
<td>0.2336</td>
</tr>
<tr>
<td>Y.Berea 7</td>
<td>2398</td>
<td>28</td>
<td>6000</td>
<td>3425</td>
<td>1733</td>
<td>0.05397</td>
<td>0.3401</td>
</tr>
<tr>
<td>Boise 8</td>
<td>2419</td>
<td>12</td>
<td>0.9</td>
<td>3593</td>
<td>1852</td>
<td>0.04957</td>
<td>0.1165</td>
</tr>
<tr>
<td>Chalk</td>
<td>2088</td>
<td>34.5</td>
<td>2.1</td>
<td>3125</td>
<td>1650</td>
<td>0.078</td>
<td>0.08197</td>
</tr>
<tr>
<td>Coal</td>
<td>1133</td>
<td>1.9</td>
<td>0.1</td>
<td>2075</td>
<td>890</td>
<td>0.2717</td>
<td>0.277</td>
</tr>
<tr>
<td>Granite</td>
<td>2630</td>
<td>0.1</td>
<td>0</td>
<td>5280</td>
<td>2903</td>
<td>0.02284</td>
<td>0.02308</td>
</tr>
<tr>
<td>Sandstone 1</td>
<td>2152</td>
<td>28.3</td>
<td>4200</td>
<td>3115</td>
<td>1411</td>
<td>0.06588</td>
<td>0.35088</td>
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<tr>
<td>Sandstone 2</td>
<td>2210</td>
<td>24.9</td>
<td>1850</td>
<td>3265</td>
<td>1641</td>
<td>0.06398</td>
<td>0.31153</td>
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We introduced the concept of effective compressibility and viewed the estimate from DARS as effective compressibility when the sample drained. Then we used dynamic diffusion to model flow in the sample. With these considerations, effective
compressibility $\kappa_c$ can be theoretically predicted by following relationship (see Appendix F for the derivation):

$$\kappa_c = \kappa_u + \frac{\phi \kappa_f}{\alpha L} \frac{e^{2\alpha L} - 1}{e^{2\alpha L} + 1},$$

where $\kappa_u$ is undrained compressibility (see explanation below), $\phi$ is sample porosity, $\kappa_f$ is fluid compressibility; $\alpha = \sqrt{i \omega / D}$, and $L$ is sample length (see Figure 2.10). Here $i = \sqrt{-1}$, $\omega$ is angular frequency, $D = k / \phi \eta \beta$ is diffusivity, $k$ is sample permeability, $\eta$ is fluid viscosity, and $\beta$ is the compressibility factor involving both the fluid and the solid matrix simultaneously. If we use DARS to measure $\kappa_c$ and $\kappa_u$, then equation (2.6) can be solved for $\alpha$ and $D$. When $\phi$, $\eta$ and $\beta$ are known from independent measurements, permeability can be obtained from $D$.

An experimental procedure was designed to measure $\kappa_c$ and $\kappa_u$ with DARS, and then for permeability estimation through equation (2.6). Figure 2.10 shows the procedure sample preparation for this experiment. Terms “drained” and “undrained” refer to the sample surface boundary conditions. In the drained condition, the sample has a partially or fully open flow surface, so that fluid can freely flow across the boundary during the DARS measurement. In the undrained condition, the sample surface is sealed or fully closed and no flow crosses the sample surface. Using drained and undrained samples, $\kappa_c$ and $\kappa_u$ can be obtained from two DARS measurements. Table 1.3 shows the permeability estimate with DARS for 17 rock samples. Table 1.3 has also presents comparisons between the DARS permeability and standard gas-injection permeability for all 17 samples. The cross-plot in Figure 2.11 is the graphical comparison of two measurements. Results from the two different methods are consistent and demonstrates how the DARS acoustics measurement can be used to estimate permeability. Future work in this area is to compare permeability to acoustic attenuation and to compare acoustical properties with changes in saturation.

![Figure 2.10: Sample surface boundary configuration. Undrained sample has a completely sealed surface. Drained sample has its cylindrical surface sealed and its two ends open. The sealing material is epoxy resin.](image-url)
Table 1.3: Permeability of 17 rocks estimated by DARS and direct gas injection, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$k_{\text{gas}}$ (mD)</th>
<th>$k_{\text{DARS}}$ (mD)</th>
<th>$(k_{\text{gas}} - k_{\text{DARS}})/k_{\text{gas}}$</th>
</tr>
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<tr>
<td>SSA4</td>
<td>361</td>
<td>335</td>
<td>7%</td>
</tr>
<tr>
<td>SSB7</td>
<td>2747</td>
<td>2754</td>
<td>-0.3%</td>
</tr>
<tr>
<td>SSC5</td>
<td>0.8</td>
<td>0.7</td>
<td>12%</td>
</tr>
<tr>
<td>SSF2</td>
<td>2669</td>
<td>2762</td>
<td>-3%</td>
</tr>
<tr>
<td>SSG1</td>
<td>1862</td>
<td>1659</td>
<td>10%</td>
</tr>
<tr>
<td>Chalk3</td>
<td>1.08</td>
<td>1.12</td>
<td>-3%</td>
</tr>
<tr>
<td>YB3</td>
<td>181</td>
<td>170</td>
<td>6%</td>
</tr>
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<td>BEN28</td>
<td>1149</td>
<td>1070</td>
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<td>BIP14</td>
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</tr>
<tr>
<td>CAS17</td>
<td>5</td>
<td>3.1</td>
<td>38%</td>
</tr>
<tr>
<td>COL25</td>
<td>0.7</td>
<td>0.05</td>
<td>93%</td>
</tr>
<tr>
<td>FEL37</td>
<td>9</td>
<td>4.5</td>
<td>50%</td>
</tr>
<tr>
<td>NIV45</td>
<td>8055</td>
<td>7240</td>
<td>10%</td>
</tr>
<tr>
<td>QUE10</td>
<td>2194</td>
<td>1950</td>
<td>11%</td>
</tr>
<tr>
<td>UNK51</td>
<td>0.9</td>
<td>0.1</td>
<td>89%</td>
</tr>
<tr>
<td>VIF02</td>
<td>12809</td>
<td>9009</td>
<td>29%</td>
</tr>
</tbody>
</table>

Figure 2.11: Comparison of the permeability of 17 samples estimated from DARS drained measurement and measured by direct gas injection.

In summary, we have successfully applied our proof-of-concept DARS system for compressibility and permeability measurements. For the application to CO2 monitoring, the effects of CO2 saturation on rock properties must be studied. Next, we will design a more advanced system based on the experience gained from current DARS. The new DARS should be able to measure a porous sample with different CO2 saturation under different effective pressures and temperatures.
2.5 A Field Example of Coal Bed Methane

We have investigated the seismic rock properties using lab measurements and rock physics models in previous subsections. We have also been involved in a field seismic monitoring study on coal bed methane (CBM) production. The coal bed is an important option for CO2 sequestration. From this field study, we have gained the knowledge on how certain seismic rock properties change with gas saturation and effective pressure in coal beds. We here present the main result from view of seismic rock properties and then give more details of this monitoring study from different views in Appendix E.

This CBM project was conducted in Powder River Basin, Wyoming. The goal of this project is to boost the CBM production by a dewatering process that reduces the pore pressure and increase gas saturation in the coal bed. Three crosswell seismic surveys were acquired in 2002, 2003 and 2004 to monitor the subsurface formation changes caused by pumping out the water (or dewatering) from coal beds. Figure 2.12 shows the zero-offset data of the time-lapse crosswell surveys. The red makers in this figure are picked from Survey 1 and then projected into Surveys 2 and 3. We use them to compare the direct $P$-wave travel times visually and then infer the trend of $P$-wave changes.

![Figure 2.12: A display of zero-offset sections for the three data sets showing evidence of dewatering-induced changes within the coal bed and repeatability of events outside the coal bed.](image-url)
Figure 2.13: $P$-wave velocity in saturated coal as a function of differential pressure.

It can be seen from Figure 2.12 that the direct $P$-wave travel time within the coal zone changes from survey to survey. The first repeated survey of 2003 shows longer travel time than the baseline survey of 2002, which means the $P$-wave velocity is reduced during this dewatering period. However, the second repeated survey of 2004 shows shorter travel time than the first repeated survey and similar travel time as the baseline, which means the $P$-wave velocity increases and goes back the baseline velocity during this dewatering period.

Figure 2.13 gives a possible explanation on these interesting up-down changes in velocities. The $P$-wave velocity changes are caused by a mixing effect of gas saturation and pore-pressure changes. At the beginning of the dewatering, the gas saturation increasing is dominant and reduces overall velocity. At later time, the dewatering mainly increases differential pressure. Differential pressure is defined as confining pressure minus pore pressure. Higher differential pressure causes the closing presumably layer cavities or air-filled cracks[34] in the coal, which makes the wave travel faster. This is may explain the $P$-wave velocity increase from Survey 2 to Survey 3. The tomogram shows the $P$-wave changes caused by the gas saturation is about 5% (see Appendix E).

3. Concepts for Continuous Monitoring

We expect the storage site to experience 3-4 distinct phases of operations: Site characterization, injection and post-injection, and closure as illustrated in Figure 3.1. Moreover, monitoring must be planned for the varying needs of these different phases and must adapt to unpredictable reservoir conditions. The cost of monitoring should decrease with time and eventually go to zero as illustrated, though the details will vary from one site to another. Moreover, the subsurface response to CO$_2$ is never fully
predictable; therefore, in order monitor the pathways properly, we may need to adjust the data acquisition configuration from time to time.

**Figure 3.1:** Three phases of a storage project. The monitoring costs should decrease with time. The cost curves (red and purple) represent two possible procedures.

Monitoring builds from the high-resolution baseline model that is developed during the characterization phase. An example of this can be seen in the Sleipner images shown in Figure 3.2. The Sleipner field, located offshore Norway in the North Sea, is considered to be the first pure sequestration project in the world. The Saline Aquifer CO₂ Storage project (SACS) is responsible for monitoring the injection. SACS has almost exclusively used seismic monitoring and produced these high quality time-lapse images that highlight changes associated with the presence of CO₂. Two subsurface images and their difference are shown in Figure 3.2. The outline of the CO₂ plume is clearly seen in addition to details believed to be CO₂ below shale barriers. These images are good examples of (relatively) high spatial resolution but low temporal resolution. Moreover, the images provide excellent qualitative snapshots of the reservoir changes but poor quantitative estimates of the magnitudes of the changes. Although the stratigraphic details seen in these images provide information about storage, they are believed to be of much less interest for the containment assessment and the “safety case” that will be required at a typical storage site.

A monitoring strategy must contribute to the safety case that will be required for licensing the site. Such monitoring activities are expected to be most active during the injection phase, decline during post-injection, and possibly end altogether during the closure phase as illustrated by the solid red line in the figure above. If, however, leaks are detected or suspected, monitoring resources must be available to address identification and mitigation of possible containment problems, as illustrated by the dash red line in Figure 3.1. In fact, closure may be defined by the significant curtailment of monitoring activities. For these reasons, monitoring serves two important purposes: (1) provides advance warning of imminent problems such leaks; and (2) confirms predicted reservoir behavior and provides data for optimizing efficiency. To this end, a practical subsurface monitoring plan should utilize information from other sources such as predictions from flow simulators and observations from wells.

These monitoring objectives can be met by combining two complementary types of subsurface imaging: (1) Reconnaissance surveys yield high temporal resolution for daily
or weekly updates; (2) Easily deployed high resolution surveys in response to reconnaissance results. These two imaging concepts can be implemented by embedding the sources and detectors in and about the storage area with surface-based and borehole-based instrumentation. The reconnaissance surveys for leak detection would be implemented, using sparse spatial sampling, to produce high temporal resolution, i.e., daily or weekly, but low-resolution spatial resolution. Circular and cross arrays are proposed as examples of sparse observation systems. If unexpected changes are detected in, existing sources and detectors already embedded can be rapidly and easily activated to produce a high spatial resolution image of the suspected region. Such a monitoring strategy must take advantage of the temporal evolution of the subsurface model and apply temporal regularization and other algorithms to improve the quality of the images generated from sparse datasets. Furthermore, by embedding the sources we can employ low-power transducers that radiate either continuous wave signals or coded waveforms as illustrate in Figure 3.3. These concepts provide numerous opportunities and challenges in imaging research, e.g., sparse data imaging, methodologies for dynamic imaging, incorporation of flow predictions, and continuous-wave signal processing. Before applying the proposed concepts to the field tests, we must perform numerous simulation studies to test their strength and weaknesses. Some of these simulation results are presented in the following sections.

**Figure 3.2:** Time-lapse seismic data from SACS yield high-resolution but qualitative difference images.
4. Special Acquisition Configurations for Continuous Monitoring

We have described the basic concepts for continuous monitoring in the previous section. Here we explore five special seismic acquisition geometries that have the potential for continuous subsurface monitoring. These proposed observation systems use seismic detectors and low-powered sources permanently embedded in the near surface and in boreholes in order to reduce operation costs and improve repeatability. Simple and composite configurations are made with acquisition units that can be adaptively added and removed during the monitoring phase. We present these special acquisition configurations in this section, and then perform computer simulations in next two sections.
4.1 Circular Array

To monitor CO₂ injection in a 3-D storage formation, we can use 2-D arrays on the surface to acquire seismic reflection data. With such arrays the reflection data can be processed to produce a 3-D subsurface image. Figure 4.1(a) shows a circular array surrounding the injection borehole. Overlapped red dots and blue small circles represent sources and receivers, respectively. This simple 2-D array configuration can record reflection seismic data for full 3-D subsurface imaging as shown in Figure 4.1(b) where reflection points corresponding to this acquisition geometry are displayed. It can be seen that the reflection coverage is very good, and a subsurface image can be obtained within the entire circle, though it will be a low-fold survey. Because the data are continuously acquired, the low-fold spatial coverage can be compensated by the abundant time-lapse data if special imaging algorithms are developed for sparse monitoring data [e.g., 42, 43, 44].

We can also use multiple circular arrays to improve the fold and subsequent image quality. Figure 4.2(a) illustrates a case of two arrays. The total reflection coverage shown in Figure 4.2(b) results from two independent arrays plus the cross recording between the two arrays. This configuration is flexible and we could add as many circular arrays as we need to reach a trade-off between cost and image quality. In geological CO₂ sequestration, the storage volume grows as the injection continues. With this configuration, we can adaptively add more arrays to track the CO₂ plume growth.

![Figure 4.1: (a) A circular array around an injection borehole; (b) the coverage of reflection points.](image)
4.2 Simple Cross Array

A simple cross array (or cross-spread) is shown in Figure 4.3. The source array (red) and the receiver array (blue) are orthogonal. The shaded area shows the reflection points covered by this acquisition geometry. This simple configuration provides a low-fold true 3-D seismic survey [27, 28] as well, but the coverage is reduced in comparison to the circular array. This sparse 3-D survey does not yield very high resolution, but it may meet the requirements for CO2 sequestration monitoring. Wapenaar [29] presented an analysis on the resolution and amplitude behavior of prestack migration using cross array data. Bouska [30] and Al-Ali [31] provided some field examples on the resolutions of sparse 3-D data.

Figure 4.3: Simple cross array. Red line is a source array; blue line is a receiver array.

4.3 3-D Cross Array

In a CO2 storage project, at least one injection borehole is available. Taking the advantage of using this borehole for monitoring, a vertical detector array may be emplaced in this borehole (Figures 4.4 and 4.5). Data recorded in the borehole are Vertical Seismic Profiles (VSP). The combined surface data and the VSP data improve the image quality. VSP data itself can be used to detect the top of CO2 plume with higher accuracy. VSP data also has richer wavefields, including both upgoing and downgoing events that provide different views of the targeted zone. Typical VSP data contains high-quality S-waves as well as P-waves. Because P-wave and S-wave data have different
responses to CO₂ saturation and pressure, using both provides a means of distinguishing saturation changes from pressure changes.

\[
\begin{align*}
&Y \\
&X \\
&Z
\end{align*}
\]

**Figure 4.4:** 3-D cross array. A vertical receiver array is added to the surface cross array of sources (red) and detectors (blue).

\[
\begin{align*}
&\text{(a)} \\
&\text{(b)}
\end{align*}
\]

**Figure 4.5:** The surface cross array records reflection seismic data (a). The vertical receiver array record both reflection and transmission data (b).

4.4 Composite Configurations

We may add more source and receiver arrays and use a composite configuration as shown in Figure 4.6. Al-Ali [31] discussed a similar configuration and used a field data set to illustrate the advantages of this configuration. Once again, we propose to add a vertical receiver array to several primary surface arrays. With this geometry, high quality 3-D VSP data will be recorded too. The combined datasets provide significant data for high performance time-lapse monitoring.
Figure 4.6: 3-D Composite configuration. In addition to multiple cross arrays, one more source arrays are placed parallel to the receiver arrays.

4.5 Adaptive Configurations

The seismic monitoring procedure may last for many years. During this long time period, the CO$_2$ plume may change dynamically, and pathways are not fully predictable. We should adaptively reconfigure the observation system as illustrated in Figure 4.7. Simple cross arrays are used as elements in this illustration.

Figure 4.7: As the CO$_2$ plume grows, the array is moved or more arrays are added to track the plume front.
5. Full-wave Seismic Simulation

To examine the realistic seismic survey, we must simulate the full seismic wavefield with realistic frequency content, realistic rock properties, a realistic source and receiver geometry. For this purpose we developed 3-D modeling codes that capture increasing complexity and more and more realistic physics of seismic wave phenomena. The suite of modeling codes is described in Appendices B and C. One of modeling algorithms in particular, a 3-D finite difference (FD) method is used here to simulate the various acquisition configurations for our monitoring strategy. Synthetic data calculated in this simulation study will be used to see if the CO2 in the subsurface is detectable. As listed in Appendix B, we have five 3-D FD programs for different media. In this section, we use the acoustic FD program for the array simulation. The acoustic wave field is the simplest yet it captures the wave behavior relevant to the array study. The models developed in the project may be used to carry out more complicated simulations that include P-waves, S-waves, and attenuation.

5.1 Model and Data

Assume that we have a storage project as shown in Figure 5.1. A layered 3-D grid model with a fault is created according to the geological model shown in Figure 1.2. We place 107 receiver arrays on the surface and one receiver array in the injection well. There are total 428 sources in four source arrays placed on the surface. At first, we calculate relatively dense 3-D seismic data. Then during data analysis, we extract different subsets from the original data volume to test different acquisition configurations.

The CO2 injected into the formation would cause P-wave velocity decrease. The velocity changes will result in travel time, amplitude and reflectivity changes in seismic waves. These seismic attribute changes are the observed data that will be used to image the CO2 storage site. To simulate different injection stages, we create four time-lapse models shown in Figure 5.2. The CO2 plume grows with time. The objective of monitoring is to track the plume front and detect possible leaks along the fault.

The FD program was run on the CEES computer cluster at Stanford Center for Computational Earth and Environment Science. In FD simulation, we had to run the program for each shot. The CPU time with an Opteron processor is 75 minutes for one shot. There are five models and 428 shots for each model. The total CPU time is 2,675 hours or 111 days, thus requiring 64 CPUs from the CEES cluster. The actual running time of the simulation was ~3 clock days. The file size of the simulated seismic data is 168 GB.
Figure 5.1: The model used for the FD simulation. The fault in this model will be used to test the leakage detection. Blue lines are receiver arrays. There are 107 surface receiver arrays and 1 vertical receiver array in the injection borehole. Red lines represent source arrays. Three of the source arrays are orthogonal to the receiver arrays, and one is parallel to the receiver arrays. Grid sizes of this model are $N_x = N_y = 361$ and $N_z = 271$. P-wave velocities, from top layer to bottom layer, are 3500, 3700, 3900, 4500, 3800, 4300 m/s. Densities are 2150, 2180, 2200, 2230, 1800, and 2200 kg/m$^3$. Ricker wavelet with a center frequency of 50 Hz is used in this FD computation.

Figure 5.2: Time-lapse CO$_2$ injection models, $M_1$, $M_2$, $M_3$, and $M_4$. CO$_2$ saturated area (red color in the center) has a velocity 3% less than the background velocity.
For a quick check on our big data volume, we extract two orthogonal zero-offset profiles form the data. Figure 5.3 shows the zero-offset synthetic data calculated from baseline model \((M_0)\). The reflections in the profiles have similar shapes as the interfaces seen in the model. Figure 5.4 shows the amplitude differences between two time-lapse surveys. The CO2 plume is clearly seen from the data display.

**Figure 5.3:** Two zero-offset gathers extracted from the whole data volume of the baseline survey \((M_0)\). Five reflections correspond to interfaces in the model. Diffractions from the fault can also be seen.

**Figure 5.4:** Amplitude differences between baseline and time-lapse surveys. (a) Difference between \(M_1\) and baseline; (b) difference between \(M_2\) and \(M_1\); (c) difference between \(M_3\) and \(M_2\); and (d) difference between \(M_4\) and \(M_3\).
5.2 An Example of Cross Array

We now extract a subset from the simulated data and study the possibility to monitor CO$_2$ storage and detect leaks using a cross array. Figure 5.5 shows a cross array that is one of the special configurations discussed previously in Section 3. The cross array is a simple layout that can acquire true 3-D seismic data.

![Cross Array Diagram](image)

**Figure 5.5:** A simple cross array used for the CO$_2$ monitoring. This configuration is the basic 3-D seismic acquisition geometry. The red line is the source array and the blue line is the receiver array.

The cross array data can be arranged as a 3-D volume. In this simulation, we use 107 sources and 107 receivers. The time window of each seismic trace is 1700 points. Therefore, we have a 3-D data cube with the size of 107$\times$107$\times$1700. We can inspect the data cube with different slices. Figure 5.6 shows a time slice, a common source gather, and a common receiver gather of the baseline survey. We can see five events from the data, which are reflections from five interfaces. The reflection exhibits different shapes on different slices. Reflections appear as circles on time slices and as hyperbolas on X or Y slices.

When CO$_2$ is injected into the formation, the seismic velocity would decrease slightly in the saturated volume. This velocity change causes the travel time increase and reflection amplitude changes. If the travel time change and the reflection amplitude change are observable, then these observed changes could be used to detect and monitor the CO$_2$ storage conditions. In this simulation we only investigate the reflection amplitude differences caused by the CO$_2$ saturation. The effects of the travel time change will be studied in the future.
Figure 5.6: Slices cut from the 3-D data cube calculated from baseline model ($M_0$). (a) A time slice at time $= 0.2856$ sec that is just above the fourth reflection. The CO$_2$ storage is located in the fifth layer. Circles on the time slice are reflections. Yellow lines show where X and Y slices are cut. (b) A Y slice (common receiver gather) cut in the middle of data cube. The yellow line indicates where the time slice is. X coordinates on this slice are locations of sources. (c) An X slice (common source gather). The horizontal coordinates here are locations of receivers.

The amplitude differences are obtained by subtracting a time-lapse survey from the baseline survey. Figure 5.7 shows the amplitude differences between first time-lapse survey and the baseline. Because the CO$_2$ is injected in the 5th layer just below the 4th interface, the seismic reflections from interfaces 1–3 are not affected by the CO$_2$ saturation. The amplitude change starts from reflection of 4th interface where CO$_2$ storage is. The CO$_2$ plume grows with time. Figure 5.8 is another comparison between baseline and a new survey acquired at time 2. It can be seen that the amplitude difference becomes larger due to a larger CO$_2$ storage plume.
Figure 5.7: Amplitude difference between baseline survey ($M_0$) and the first time-lapse survey ($M_1$). Seismic reflections observed on the surface can show where the top of the storage is.
Figure 5.8: Amplitude difference between baseline survey ($M_0$) and another later survey ($M_3$). As the CO$_2$ saturated area grows larger in this model, the amplitude difference becomes bigger too.

To study if CO$_2$ leaks along a fault could be detected and how the aspects of the leaks appear in the cross-spread data, we create a model as shown in Figure 5.9. CO$_2$ leaks change the seismic velocity near the fault. The velocity perturbation causes some seismic diffraction. The time slice in Figure 5.9 clearly shows the diffraction pattern related to the leaks along the fault. The special circular pattern is interesting. In the field data, the noise is always there, but it is random and should not generate anomaly like this pattern. This simulation shows a simple cross-spread may be capable to detect the possible leaks by looking for this kind of circular patterns.
Figure 5.9: Amplitude difference between baseline survey ($M_0$) and a much later survey ($M_4$). CO$_2$ starts leaking along the fault in this model. The time slice is cut at 0.2764 sec as indicated on the Y slice. A circular anomaly above where the leakage occurs can be seen from this time slice. It is also interesting to see that the circular contours have a linear discontinuity highlighted by the dashed line. This discontinuity is caused by the fault.
6. Subsurface Imaging with Simulated Data for Circular Array

In many cases, it is not enough to look only the raw data like we did in previous section. We need to use the data for surface imaging and understand the storage status directly from the subsurface images. We now use a simple model problem shown in Figure 5.1 for a subsurface imaging simulation. The model has a constant wave velocity and two reflectors. CO₂ is injected into the formation between these two reflectors. A circular array (see Section 3.1) is placed on the surface around the injection borehole. Two time-lapse surveys are simulated with the finite difference method. The CO₂ saturated area has two different sizes in these two surveys.

We apply Kirchhoff prestack depth migration to these two 3-D data sets, respectively, and obtain two 3-D subsurface depth images. The difference between two time-lapse images shows the injection activities. Figure 5.2 is a slice cut from the difference image cube at the depth where injection occurs. The difference indicated in this depth slice shows the expanding of CO₂ saturated area, which is the same as the true model. Circles appeared in this depth slice are migration artifacts due to this sparse circular acquisition geometry. We hope that multiple circular arrays and more sophisticated imaging algorithms can reduce the artifacts.

![Figure 6.1: A simple mock-up of CO₂ sequestration monitoring. The light blue square between two reflectors indicates the CO₂ injected.](image1)

![Figure 6.2: Depth slice cut from difference image cube.](image2)
7. Dynamic Imaging

In time-lapse monitoring the quantity of interest, e.g., seismic velocities, is slowly time-varying while the dynamics that govern the physics of the measurement, e.g., traveltimes, is spatial and essentially instantaneous in nature. For this reason, traditional methods ignore the temporal aspect of the problem and solve the inverse problem independently for each time instant under the assumption that the data acquisition is also instantaneous. There are other approaches [44, 45] to the time-lapse imaging. Our approach is to use sparse time-lapse datasets and incorporate the slow temporal variations into the formulation and processing of the image. We describe three dynamic imaging methods that can be used for our coupled spatio-temporal imaging problem: Kalman filter, recursive least squares and recursive re-weighted least squares, and temporal regularization.

Suppose that we want to estimate the parameter of a dynamic model \( m \) that changes with time. Let \( m_i \) be the model at time \( t_i \), \( i = 1, 2, \ldots, N \). Assume that the model parameter at \( t_{i+1} \) can be predicted with the relation of the form

\[
m_{i+1} = A_i m_i + \eta_i ,
\]

(7.1)

where \( A_i \) is a transition operator that describes the temporal evolution of the model parameter and \( \eta_i \) is the prediction error that for now is Gaussian with zero mean and covariance

\[
Q_i = E(\eta_i \eta_i^T). \tag{7.2}
\]

The prediction filter would come from a flow simulation run on the high-resolution baseline model or from an evolution model derived from temporal history of the images. A general time varying imaging problem can be described by

\[
d_i = G_i m_i + \varepsilon_i ,
\]

(7.3)

where \( d_i \) is the observed data and \( G_i \) represents the physical relation between the data \( d_i \) and the model \( m_i \). The measurement error \( \varepsilon_i \) is also assumed to be Gaussian with zero mean and covariance

\[
R_i = E(\varepsilon_i \varepsilon_i^T). \tag{7.4}
\]
7.1 Kalman Filters

If \( A_i \) is known or can be estimated, we can obtain a general solution for the dynamic imaging problem using both model predictions and data. One such solution is given by the Kalman filter\(^{[48]}\) according to following procedure:

\[
\begin{align*}
\hat{m}_0 &= \hat{m}_{\text{prior}} \\
\hat{C}_0 &= C_{\text{prior}} \\
\hat{C}_i &= A_{i-1}\hat{C}_{i-1}A_{i-1}^T + Q_{i-1} \\
K_i &= \hat{C}_i G_i^T(G_i\hat{C}_i G_i^T + R_i)^{-1}.
\end{align*}
\]

(7.5)

Here \( \hat{C}_i \) is the prediction of model covariance, \( \hat{C}_i \) is the model parameter covariance updated with the data recorded in time instant \( t_i \), \( m_{\text{prior}} \) is initial model, and \( C_{\text{prior}} \) is an initial covariance estimate. The Kalman filter possesses two significant advantages: optimality in LMS sense and recursion.

In spite of its widespread applicability, the Kalman filter still has some practical limitations. First, it requires knowledge of \( Q_i \) and \( R_i \) at all times. When these quantities are unknown, it is necessary to estimate them. In some practical applications, however, it is hard to do this, and incorrect variance estimates significantly degrade the performance of the resulting estimate. In order to overcome the resulting difficulties in applications of Kalman filtering, a recursive estimation procedure that does not require knowledge of the noise covariance is required.

7.2 Recursive Least Squares and Re-weighted Recursive Least Squares

The second method we consider is called recursive least squares (RLS). In RLS, the model parameter at time \( t_i \) is obtained by minimizing

\[
\phi = \sum_{j=0}^i ||G_jm - d_j||^2.
\]

(7.6)

Note that in equation 7.6 all the data recorded in previous time have the same weight in determining the estimate for the model \( m_i \). The solution is obtained by a recursive process:
\[ \hat{m}_0 = m_{\text{prior}} \]
\[ \hat{C}_0 = C_{\text{prior}} \]
\[ K_i = \hat{C}_{i-1} G_i^T (G_i \hat{C}_{i-1} G_i^T + I)^{-1} \]  
\[ \hat{m}_i = \hat{m}_{i-1} + K_i (d_i - G_i \hat{m}_{i-1}) \]
\[ \hat{C}_i = (I - K_i G_i) \hat{C}_{i-1} \]  

This result is identical to a Kalman filter if \( A_i = I \) and \( Q_i = 0 \) in equation 7.5.

A generalized RLS, called recursive re-weighted least squares (RRLS), can be obtained by introducing the concept of "forgetting" in which older data is gradually discarded in favor of more recent information. In least squares method, forgetting can be viewed as giving less weight to older data and more weight to recent data. The objective function is defined as

\[ \sum_{j=0}^{i} \lambda^{i-j} \| G_j m - d_j \|^2 \],  

where \( \lambda \) is called the memory or "forgetting factor" and \( 0 < \lambda \leq 1 \). The scheme is also known as least-squares with exponential forgetting. The model \( m_i \) is calculated recursively as

\[ \hat{m}_0 = m_{\text{prior}} \]
\[ \hat{C}_0 = C_{\text{prior}} \]
\[ K_i = \hat{C}_{i-1} G_i^T (G_i \hat{C}_{i-1} G_i^T + \lambda)^{-1} \]
\[ \hat{m}_i = \hat{m}_{i-1} + K_i (d_i - G_i \hat{m}_{i-1}) \]
\[ \hat{C}_i = (I - K_i G_i) \hat{C}_{i-1} \lambda^{-1} \].

We can define the memory of the algorithm as \( \mu = 1/(1 - \lambda) \). When \( \lambda = 1 \) (standard RLS) the memory is infinite, which means that all the data have the same weight in the current estimation. If \( \lambda \) approaches zero, the algorithm has little memory on the old data.

7.3 Temporal Regularization

Finally, and perhaps the simplest approach is to add temporal regularization to the conventional formulation of the inversion problem as follows. Consider again the series of \( n \) datasets \( (d_i) \) acquired at \( n \) different times \( (t_i) \) but this time with different kernels \( G_i \). In the conventional case, this inversion is solved independently for each dataset by minimizing an objective function of the form
\[
\Phi(m_1, ..., m_n) = \sum_{i=1}^{n} \| G_i m_i - d_i \|^2 + \lambda^2 \sum_{i=1}^{n} \| Dm_i \|^2, \tag{7.9}
\]

where \(D\) is a weighting operator and \(\lambda_s\) is a regularization parameter. The first term in equation 7.9 measures data misfit while second measures model length as modified by \(D\). If \(D = I\), then the resulting minimization of equation 7.9 is simply the damped least-squares solution. Neither of the terms in equation 7.9 couple solutions across multiple times.

In the case of time-lapse monitoring, models will have temporal correlation. Based on a straightforward extension of Tikhonov regularization, we modify equation 7.9 to include temporal cross-coupling which minimizes the time-lapse change in some model attribute in addition to data misfit minimization for individual datasets. Consider a combined objective function of the form

\[
\Phi(m_1, ..., m_n) = \sum_{i=1}^{n} \| G_i m_i - d_i \|^2 + \lambda^2 \sum_{i=1}^{n} \| Dm_i \|^2 + \lambda_s^2 \sum_{i=1}^{n-1} \frac{\| Dm_{i+1} - Dm_i \|^2}{\Delta t_i}, \tag{7.10}
\]

where \(\Delta t_i = t_{i+1} - t_i\), and \(\lambda_s\) is a regularization parameter controlling the strength of the temporal constraint.

In principle, the tomography operators used in the formulations presented above can be replaced with any other imaging operator, e.g., diffraction tomography or migration. In practice, however, considerable effort will be required to implement other more complicated operators. Nevertheless, tomography is consistent with our strategy of low spatial resolution and high temporal resolution imaging.

8. Velocity Tomography with Temporal Regularization

8.1 Traveltime Formulation

As summarized above, we have multiple datasets recorded at different times. In this section, we test the approach of temporal regularization as a way to implement a joint inversion. We use a cross-well seismic geometry with synthetic and CBM field datasets to demonstrate temporal regularization. First, we note that the velocity tomography problem is highly nonlinear, but solved in linear steps of ray tracing and inversion. Linearization results in the regularized linear tomographic system

\[
\begin{align*}
G_i \delta m_i &= \delta \tau_i, \\
\lambda_s S \delta m_i &= 0 \quad i = 1, ..., n, \\
\lambda_s T \delta m &= 0
\end{align*}
\tag{8.1}
\]
where $\delta t_i$ are vectors of residuals of the traveltimes for each of the $n$ time-lapse surveys, $\delta m_i$ are the perturbations of the slowness to be determined, $\delta m=[\delta m_1, \delta m_1, \ldots, \delta m_n]$ and $G_i$ are the tomographic matrices associated with each of the time-lapse datasets and models. A schematic outline of the algorithm for temporal regularization is illustrated in Figure 8.1. The problem is to minimize the combined objective function of the form

$$
\Phi(m_1, \ldots, m_n) = \sum_{i=1}^{n} \| G_i m_i - d_i \|_2^2 + \lambda_s^2 \sum_{i=1}^{n} \| Dm_i \|_2^2 + \lambda_t^2 \sum_{i=1}^{n-1} \frac{\| Dm_{i+1} - Dm_i \|_2^2}{\Delta t_i},
$$

(8.2)

where $\Delta t_i = t_{i+1} - t_i$, and $\lambda_s$ is a regularization parameter controlling the strength of the temporal constraint.

**Figure 8.1**: Outline of a temporal regularization algorithm
Minimizing equation 8.2 is equivalent to the least-squares solution of an augmented system

\[
G \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_n
\end{bmatrix} = \begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
0
\end{bmatrix}
\]

with

\[
G_c = \begin{bmatrix}
G_1 & 0 & \cdots & 0 & 0 \\
0 & G_2 & \cdots & 0 & 0 \\
\vdots & 0 & \ddots & 0 & 0 \\
0 & \vdots & 0 & 0 & G_n \\
\lambda_s D & 0 & \cdots & 0 & 0 \\
0 & \lambda_s D & \cdots & 0 & 0 \\
\vdots & 0 & \ddots & 0 & 0 \\
0 & \vdots & 0 & 0 & \lambda_s D \\
\lambda_r D / \Delta t_1 & -\lambda_r D / \Delta t_1 & \cdots & 0 & 0 \\
\vdots & 0 & \ddots & 0 & 0 \\
0 & 0 & \cdots & \lambda_r D / \Delta t_{n-1} & -\lambda_r D / \Delta t_{n-1}
\end{bmatrix}
\]

This formalism also provides a good approach to incrementally acquired seismic surveys where the survey geometry at any particular time step \(n\) is relatively sparse. Although independent inversion of a single survey might yield an image with very low spatial resolution, by jointly inverting a series of surveys we can effectively add spatial aperture in exchange for losing temporal resolution.

To this point, our formulation has been relatively general with no assumption regarding the operation which \(G\) performs, the model parameterization represented by \(m\), or the type of data stored as \(d\). We will now apply our formulation to the concrete example of seismic traveltime tomography with one temporal dimension and two spatial dimensions. In this case we choose each \(m\) to be a rectangular mesh of homogeneous slowness cells while \(d\) is a vector of picked first-arrival traveltimes and \(G\) is the ray-path
matrix. We use a split Laplacian operator \((D_{xx}, D_{zz})\) to allow anisotropic regularization with two spatial parameters \((\lambda_{sx}, \lambda_{sz})\) and two spatiotemporal parameters \((\lambda_{tx}, \lambda_{tz})\) for the respective terms in equation 8.5. Regularization parameters are chosen by observation although use of the L-surface technique advocated by Brooks et al. [46] would decrease the amount of manual tuning required in the inversion process. The resulting coupled systems were solved using the LSQR algorithm [47].

8.2 A Synthetic Example

For this synthetic experiment we generate four time-lapse models of a CO₂ flood progressing through a permeable layer shown in Figure 8.2(a). Data are synthesized for a cross-well geometry with 40 sources and 40 receivers evenly spaced near the right and left boundaries of the model, respectively. Gaussian noise (~3%) is added to the traveltime picks for all four synthetic surveys. The first survey is the baseline that has full coverage. Three repeated surveys only have partial coverage as illustrated in Figure 8.2(b). Figure 8.3 shows the difference images reconstructed without using any temporal constraint. In this case, the time-lapse images (or tomograms) are calculated independently from each partial survey, and then compared to the baseline image. The difference images are the difference between time-lapse images and the baseline image. Clearly, these difference images are good representations of the true models. Figure 8.4 is the difference images obtained from joint inversion of multiple time-lapse datasets. It can be seen, from Figures 8.3 and 8.4, that the joint inversion with a temporal constraint has clearly improved the difference imaging results, especially for this case of using incrementally collected time-lapse datasets.

![Figure 8.2](image)

**Figure 8.2:** An incremental time-lapse acquisition example of cross-well geometry. (upper) True velocity models; (lower) data coverage at different times.
Figure 8.3: Difference images without temporal constraint. (upper) True models; (lower) temporally independent reconstructions from the independent partial surveys.

Figure 8.4: Difference images with temporal constraint. (upper) True models; (lower) joint temporally constrained reconstructions from time-lapse surveys.
Most traditional time-lapse processing techniques involve the solution of equation 8.1 for each of the independent measurements and the temporal variations are obtained by subtracting the images. Naive image subtraction tends to be sensitive to survey-to-survey changes in S/N ratio and variations in acquisition geometry, both of which can generate artifacts in the resulting time-lapse images and cross-equalization algorithms [49] are needed to match the geometry and signal characteristics of repeated surveys. We believe that taking into account the temporal aspects during the time-lapse processing can improve the coherence between the images and may consistently counter irregular acquisition geometries. In addition to fit the data and the spatial constraints, the additional constraints imposed by temporal regularization are imposed on the solution.

8.3 Field Data Example

We applied this spatio-temporal regularization scheme to coalbed methane production monitoring. The observed data are three independent cross-well surveys recorded for the Big George coal in the Powder River Basin (WY) at different phases of methane production. The three resulting tomograms are shown in figure 8.5. We used the first derivative operator for spatial regularization and the temporal regularization operator is the difference between the model derivatives of consecutive surveys. There is a clear though subtle decrease in velocity in the coal layer (1100 ft – 1200 ft) from the baseline to the second survey and then an increase in velocity in the third survey. These changes are easily visible in Figure 8.6 where the difference tomograms with respect to the baseline survey are shown. Notice that artifacts outside the low velocity coal present but considerably reduced by spatio-temporal regularization. The initial velocity decrease in the coal is caused by methane desorption. The subsequent increase in velocity is due to bulk hardening of the coal as pore pressure is further reduced due to furthering dewatering. This result is consistent with the coal physics model discussed in Section 2 above.

![Figure 8.5: Time-lapse velocity tomograms obtained through spatio-temporal regularization. (left) baseline; (middle) after 6 months; (right) after 15 months](image)
Figure 8.6: Percent difference between the spatio-temporal regularized result shown in figure 8.5. Notice that the velocity decreased coal layer (1100-1200 ft) between survey 1 and 2, but then increased and became almost equal to the baseline velocity.
9. Conclusions

Our studies have shown that geological CO2 sequestration can be monitored by various geophysical methods, though seismic seems is most suitable because it is applicable to most any geological setting. While it would be very difficult to accurately predict the absolute values of the seismic parameters at a specific site either before or during CO2 injection, a reasonable estimate of the time-lapse changes is more likely given a high-resolution baseline model. Moreover, because we are most interested in monitoring CO2 containment, we need not to use regular and expensive 3-D seismic surveys designed to detect detailed geo-stratigraphy or structure of the CO2 plume. Moreover, subsurface CO2 demands periodic assessments at time intervals of weeks or days, not years as ordinarily used in petroleum applications. This can be accomplished only with a new paradigm for seismic imaging. On the basis of these considerations, we suggest a tradeoff between 3-D spatial resolution and temporal resolution. This tradeoff may be accomplished through use of innovative data acquisition and data processing strategies discussed in this report. For example, four 3-D sparse data acquisition configurations are proposed, along with a data processing sequence that leads to continuous monitoring. Full wave field modeling was used to investigate these special acquisition and processing algorithms. The simulated results are encouraging. Field tests are needed to better understand their strengths and weaknesses.

A new laboratory method, DARS, was used to investigate the acoustical properties of porous rocks at low frequency. A coincidental finding of this research was that the DARS technique is useful at estimating the permeability of porous materials. Data from a coalbed seismic monitoring project was used to test models for changes in seismic velocity with pressure and gas saturation. These in situ observations confirmed the changes predicted by theory and that the observations can be reliably made at field scale. This observation gave us some useful information for future CO2 monitoring in coal beds.
10. Publications

1. Xu, Chuntang, Jerry M. Harris, and Youli Quan, 2006, Estimating flow properties of porous media with a model for dynamic diffusion, SEG Expanded Abstract.
11. References

40. Xu, Chuntang, Jerry M. Harris, and Youli Quan, 2006, Estimating flow properties of porous media with a model for dynamic diffusion, SEG Expanded Abstract.
12. Appendices

Appendix A - Feasibility Study on Geophysical Monitoring of CO2 Sequestration

Various geophysical methods may be used for monitoring CO₂ sequestration. We here investigate the applicability of each method, except the seismic method that has already been studied in Section 1. Rock physics models are used to determine the time-lapse changes in relevant physical properties (e.g., acoustic, electrical, gravity, and geodetic) for a variety of rock types. These rock physics models are used in a synthetic formation model to estimate field scale changes. Results from different settings are compared to suggest optimum monitoring techniques for monitoring geologic sequestration. Seismic, electromagnetic, gravitational, and geodetic methods are the four broad types of subsurface geophysical monitoring examined. As a conclusion, the seismic method seems most suitable for the CO₂ sequestration monitoring. Table A-1 is a summary of those geophysical methods discussed in this section.

Table A-1: Summary of the usefulness of geophysical techniques by use and setting.

<table>
<thead>
<tr>
<th></th>
<th>Seismic</th>
<th>Electromagnetic</th>
<th>Gravity</th>
<th>Deformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂ Migration</td>
<td>good</td>
<td>good</td>
<td>poor</td>
<td>Poor</td>
</tr>
<tr>
<td>Leak Detection</td>
<td>good</td>
<td>good</td>
<td>poor</td>
<td>no</td>
</tr>
<tr>
<td>Geologic Setting</td>
<td>any (no gas)</td>
<td>aquifers</td>
<td>any</td>
<td>oil and gas</td>
</tr>
<tr>
<td>Mass Balance</td>
<td>poor</td>
<td>poor</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>Rock Strength</td>
<td>any (soft better)</td>
<td>any</td>
<td>any</td>
<td>soft</td>
</tr>
<tr>
<td>Formation</td>
<td>any</td>
<td>any</td>
<td>shallow</td>
<td>shallow</td>
</tr>
<tr>
<td>Depth</td>
<td></td>
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<td></td>
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</tbody>
</table>

A-1 Background

There is a wide range of monitoring techniques available to monitor CO₂ sequestration. These methods range from spaceborne satellites to surface stations and borehole devices. The two general options for monitoring are direct and remote sensing methods. Direct sampling methods have high spatial resolution but low spatial coverage; examples of these are surface chemical sensors and monitoring wells. The techniques discussed here are subsurface geophysical imaging techniques, which generally have high spatial coverage but low spatial resolution. These are divided into seismic, electromagnetic, gravitational, and geodetic techniques. Geophysical methods have the added benefit of being remote. While a monitoring well would have to penetrate the formation seal to gather meaningful hydrologic data or fluid samples, possibly creating conduits for CO₂ to escape, geophysics may be used to image the area of interest without such intrusion.
In examining our monitoring goals, it is evident that none of those goals may be wholly achieved without the aid of geophysics, nor will geophysics alone provide the solution we need. Geophysical methods will be necessary to assess CO₂ movement and storage for a high spatial coverage that monitoring wells alone will be unable to provide. That is not to say that direct methods will not be useful; a combination of direct and remote techniques will certainly be necessary to effectively monitor sequestration.

As opposed to the use of geophysics for characterization, where the subsurface geology is unknown, time-lapse monitoring is only carried out after extensive characterization has been carried out in baseline surveys. Repeatability then becomes an important issue, which may be solved through the use of fixed measurement devices either on the surface or in the subsurface. The benefit of time-lapse monitoring is that some effects, such as lithology and cementation, are removed as they are assumed to remain constant between surveys [1]. The change is then associated only with changes in the pore fluid composition and pore pressure. Most of the research occurring in this area has been in seismic reflection and tomography, while geodetic techniques, also time-lapse, have seen less use in subsurface monitoring.

The most significant changes in the properties of the rocks and fluids in a formation undergoing CO₂ injection are expected to result from saturation and pressure changes. Pressure and saturation changes may have a dramatic effect on the bulk properties of the fluid. Increasing CO₂ saturation will cause the bulk fluid density and viscosity to decrease, while the effective compressibility will be dramatically increased. Brine conductivity is treated as a constant as the water salinity is assumed to have reached an equilibrium state. Isostress mixing is assumed in the modeling; mixing of the fluids in the pore space is assumed to occur at the finest scale. Changes in the pore fluid and pore pressure bring about a host of changes in the effective properties of the saturated rocks, which may be detected through the use of remote sensing techniques. The specific changes will be addressed individually, but in general there are changes in the physical, acoustic, and electrical properties, ranging from few percent changes in seismic velocity to order of magnitude changes in rock conductivity.

A-2 Formation Modeling

The four broad subsurface imaging techniques will be examined separately. Each contains a discussion of how the relevant fluid properties change with pressure and CO₂ saturation. This fluid model is then incorporated into a rock physics model, which is dependent on both rock type and pore fluid. Finally each of the rock physics models is applied to a reservoir model to produce field scale changes, which are compared for the settings of interest. The reservoir model is a cylindrically symmetric tabular reservoir 100 meters thick with a vertical injection well at its center. Injection is at a constant mass rate of 1.5 million tons per year of CO₂, and results are displayed after 10 years of injection. The saturation curve and the pressure curve are shown in Figure A-1. The vertical injection well is at the left edge of the figure. This injection model is meant to qualitatively capture the behavior of a CO₂ front, not to be a rigorous reservoir simulation.
Figure A-1: Radial profile of saturation and pressure at constant depth. The pressure is communicated beyond the saturation front.

The three aspects of CO\(_2\) front behavior that the injection model attempts to capture are gravity segregation, mixing at the saturation front, and pressure front behavior. As can be seen in Figure A-2a the bubble geometry is driven by gravity segregation resulting from the differences in fluid densities. Under most formation conditions CO\(_2\) will be lighter than the fluid in-place, resulting in a vertical gravity drive. Figure A-2b shows the pore pressure in the reservoir, which is a combination of hydrostatic background pressure with a perturbation from injection pressure varying with radial distance from the well.

Figure A-2: (a) CO\(_2\) saturation and (b) formation pore pressure in Mpa as a function of depth and radius after 10 years of simulated injection.

A-3 Seismic
A-3.1 Seismic Model

The seismic properties of the pore fluids that we are concerned with are density and the bulk modulus. The properties of the brine and oil initially present in the formation are fairly insensitive to reservoir conditions while the seismic properties of CO\(_2\) are much stronger functions of pressure and temperature (Figure A-3). We use the relations collected by Batzle and Wang [2] to estimate the seismic properties of the fluids, e.g., oil, brine, and hydrocarbon gas. Gassmann’s fluid substitution is a low frequency theory, which allows one to determine the effect of pore fluid changes on rock moduli. Using the above effective fluid properties in Gassmann’s equation [3] along with the mineral modulus and the dry rock modulus, one can solve for the saturated moduli with
\[
\frac{K_{\text{sat}}}{K_0 - K_{\text{sat}}} = \frac{K_{\text{dry}}}{K_0 - K_{\text{dry}}} + \frac{K_\phi}{\phi(K_0 - K_\phi)} \quad \text{and} \quad \mu_{\text{dry}} = \mu_{\text{sat}},
\]

(A-1)

where \(K_0\), \(K_{\text{dry}}\), and \(K_{\text{sat}}\) are the mineral, dry rock, and saturated bulk moduli, respectively. \(\phi\) is the porosity and \(\mu\) is the shear modulus, which is unchanged upon fluid substitution under Gassmann’s theory. The saturated density also changes as a result of changing the pore fluid, and can also be calculated from

\[\rho_{\text{sat}} = \phi \rho_{\text{fl}} + (1 - \phi) \rho_{\text{dry}}\]

(A-2)

To model the stress-dependence of fractured rocks we use the results of Eberhart-Phillips [4]. Their work is based on data gathered by Han [5] on the stress-dependent velocities of 64 sandstone samples. In practice, stress-dependence will need to be determined as part of site characterization. Eberhart-Phillips used only sandstone data, but a similar stress-dependence may occur in fractured carbonates. They found the following empirical relation for compressional and shear velocity as a function of porosity, clay content, and effective pressure:

\[
V_p = 5.77 - 6.94\phi - 1.73\sqrt{C} + 0.446(P_e - e^{-16.7P_e})
\]

(A-3a)

\[
V_s = 3.70 - 4.94\phi - 1.57\sqrt{C} + 0.361(P_e - e^{-16.7P_e})
\]

(A-3b)

In these two expressions, \(C\) is the mineral fraction of clay, \(P_e\) is the effective pressure in kbar and \(V_p\) and \(V_s\) are in km/s. Figure A-4 illustrates the effect of changing effective stress on a particular sample, StPeter1. These data were collected for water-saturated rocks.

![Figure A-3](image_url)

**Figure A-3:** Compressional velocities in (a) brine and oil, and (b) CO₂ as a function of pressure and temperature.
Combining Gassmann and Eberhart-Phillips allows one to predict the changes from increasing pore pressure and changing saturation with injection and compare the two effects. Figure A-5 and Figure A-6 display the results of numerical experiments on a stress-dependent sandstone and a stiffer, unfractured carbonate undergoing CO₂ flooding. The top curve in each plot is Gassmann, while each of the other curves assumes a linear increase in pressure with CO₂ saturation. Each curve begins at the same reference pore pressure and at zero CO₂ saturation. The first thing to notice is that the stiffer rock has a much smaller percent change in velocity, meaning that any changes will be much harder to detect. Also important is that in the fractured sandstone approximately half of the compressional velocity change results from saturating changes and half from pressure effects, while the shear velocity is more affected by pressure changes, which agrees with published results [6].

Figure A-5: Calculated (a) compressional and (b) shear velocities with CO₂ saturation using Gassmann fluid substitution and sandstone with stress dependence.
**Figure A-6:** Calculated (a) compressional and (b) shear velocities with CO₂ flooding using Gassmann fluid substitution for a stiff unfractured carbonate rock with stress dependence.
A-3.2 Lab Data

Wang and Nur [7] conducted laboratory experiments on sandstone samples under hydrocarbon saturated and CO2 flooded saturations. The samples were initially saturated with n-hexadecane then flooded with CO2 leaving approximately 30% residual oil. The confining stress was kept constant at 20 MPa while the pore pressure was increased from approximately 0 to 18 MPa. The results for the Beaver No.7 sample are shown in Figure A-7a. Figure A-7b shows the simulated results from our model.

The compressional velocities display similar qualitative behavior while the shear velocities exhibit some striking differences. From Gassmann theory we predict that the shear modulus is unchanged upon flooding, and any velocity change will be the result of density changes. As less dense CO2 is displacing hydrocarbon oil we expect that flooding will always increase shear velocity. The unexpected behavior of the shear velocity curves in the lab data can be attributed to high frequency viscous effects; Gassmann is a zero frequency equation and cannot always describe sample behavior at laboratory frequencies. Measurements made at field frequencies are expected to show more Gassmann like behavior.

![Figure A-7: A comparison between (a) lab data from Wang [5], Beaver No. 7 and (b) our stress-dependent fluid substitution model. Black lines are isotherms for hydrocarbon saturated rocks and blue lines are isotherms for CO2 flooded rocks. Confining pressure for all plots is 20 MPa.](image)

A-4 Electromagnetic

While not as popular as seismic methods in the oil industry, electromagnetic (EM) techniques have much to offer in the area of monitoring sequestration. The expected changes in electric and magnetic properties to be measured with electromagnetic techniques, most notably conductivity, may be of an order of magnitude or more, as compared to seismic methods where changes are typically on the order of a few percent. This is not to say, however, that electromagnetic techniques will be more useful than seismic techniques in CO2 sequestration. Steel casing severely attenuates higher frequency electromagnetic signals, and so reduces the resolution that may be attained. Additionally, common earth materials may vary in conductivity by as much as six orders of magnitude, so detecting an order of magnitude change may prove challenging.
Nonetheless, electromagnetic monitoring offers us the ability to measure CO₂ saturations and provides a complimentary set of measurements to seismic.

A-4.1 Electromagnetic Model

In dealing with field scale electromagnetic measurements, conductivity plays a dominant role in electric and electromagnetic techniques. Rock conductivity is very sensitive to brine saturation and brine conductivity, which in turn depend on the salinity and temperature of the brine (Figure A-8). CO₂ and other types of initial reservoir fluids are very resistive and have a negligible impact on the bulk conductivity of both the fluid and the rock. Here we will estimate brine resistivity at 18°C using a polynomial fit [12] then make an approximate temperature conversion [13].

**Figure A-8:** Brine resistivity in ohm meters as a function of temperature and salinity.

Rock conductivity may be estimated with Archie’s Law and its various modifications. It is an empirical formula, which must be fit to the reservoir rocks in the area of interest. In its basic form, Archie’s law is given by

\[
\sigma = \frac{1}{a} \phi^m S_w^n \sigma_w. \quad \text{(A-4)}
\]

This expression relates the bulk conductivity \( \sigma \) of the rock to the porosity \( \phi \), water saturation \( S_w \), and water conductivity \( \sigma_w \). \( a, m, \) and \( n \) are dimensionless constants which will need to be determined for a particular formation, with typical values for clean sandstones around 1, 2, and 2 respectively. Higher values of \( m \) have been reported for Middle-Eastern carbonate rocks [14].

Conductivity may be measured directly in the shallow subsurface with electrical resistance tomography (ERT). In deep reservoirs and aquifers, however, boreholes are widely separated and conductivity measurements need to be made over large distances. Using ERT in this manner produces extremely low-resolution images of the subsurface, and so other techniques have been developed for this type of situation. Modern electromagnetic techniques make use of magnetic source and receiver dipole antennas to
propagate electromagnetic waves over long distances[15]. Early work in this field involved the use of 15 and 17 MHz signals to provide high resolution images using strait ray tomographic techniques[16]. Using such high frequencies, well spacing was limited to several meters.

A-4.2 Reservoir Scale Simulations

Figure A-9 shows the electromagnetic results from our injection simulation. As there is no pressure dependence for brine resistivity the conductivity profile simply tracks the saturation profile. We see uniform conductivity in the fully flooded and unflooded regions with approximately an order of magnitude difference, which we came to expect from our rock physics model. The local attenuation profile (Figure A.9b) changes by a factor of 3 between the flooded and unflooded regions. Like seismic, detailed forward modeling will be required to take conductivity and attenuation profiles such as these and convert them to measured signals. Strait ray methods are not appropriate for low frequency measurements and only provide a first approximation of the expected attenuation.

![Figure A-9: (a) Formation conductivity in mho/m and (b) local attenuation in m⁻¹.](image)

Electromagnetic techniques are not strongly dependent on rock type, rock strength, or formation depth, but are dependent on initial and final fluid saturations. Aquifers will be the best candidates for electromagnetic monitoring as they will have the largest brine saturation changes and therefore the largest conductivity and attenuation change. Changes in $R_{CO_2}$ over time are not expected to greatly change the conductivity of the formation fluid, though there will be some additional conductivity associated with the additional ions in the fluid from the formation of carbonic acid.

A-5 Gravity

The last two techniques we will discuss are gravitational and geodetic techniques which are very similar from a modeling standpoint. The model we will be using is Newtonian gravity. Changes in pore size from increased pore pressure are expected to be negligible compared to the change in gravity resulting from fluid density changes (equation 1.2). Gravity is a low-resolution technique with fundamentally non-unique solutions. Constraining inversions with formation geometry and using only time-lapse information can result in much better results.
A-5.1 Gravity Model

Brine and oil density are relatively insensitive to changes in pressure and even to increased CO₂ in solution. Almost all of the changes in fluid density associated with CO₂ injection will be from the lower density of the CO₂. Figure A-10 shows the density of CO₂ as a function of depth with a hydrostatic pressure gradient and a typical geothermal gradient (a), and also as a function of pressure and temperature (b). It is apparent that as the formation depth increases the CO₂ density will increase to the point where there is very little density contrast between the CO₂ and the initial reservoir fluid, in which case there will be no measurable anomaly.

![Figure A-10](image)

**Figure A-10:** (a) CO₂ density as a function of pressure and temperature and (b) as a function of depth with a hydrostatic pressure gradient and a typical geothermal gradient.

The perturbation to the gravitational field due to a point source with some discrete volume dV, porosity φ, and density change Δρ, at a distance r is given by

\[
\Delta g(r) = -\phi \Delta \rho \frac{G \hat{r}}{r^2} dV
\]  

(A-5)

In general the reservoir will have a complex geometry and variable saturation due to formation heterogeneity, and as such the contributions of discrete points of density change will need to be summed to find the change in the gravitational field. This solution has the form

\[
\Delta g_i(x) = -G \int dV \phi \Delta \rho \frac{x_i - \xi_i}{|x - \xi|^3}
\]  

(A-6)

where Δg is the change in the gravitational field at position x and ξ is the spatial variable for the distribution of density changes. This expression is very similar to the deformation model (equation A-4), which will be discussed in the next chapter. Clearly, a stronger signal will result from shallower reservoirs and higher density contrasts. Porosity will have less effect as reducing the porosity will simply force the CO₂ to occupy the same pore volume, but in a larger bulk volume.

A-5.2 Reservoir Scale Simulations

We can see that the bulk density percent changes are small (Figure A-11) which is expected as the CO₂ density is about 0.6 g/cc and the bulk of the mass is in the rock.
The gravity change for our homogenous formation is shown in Figure A-12. The curves are for profiles at constant depth, and the radius is measured from the center of the injection well. For example, the measured time-lapse gravity signal after 10 years of injection directly over the injection well at a depth of 600 meters would be approximately 30 microgals, well above instrument sensitivity in the absence of cultural noise. Newer gravimeters may have resolution as low as one microgal.

Figure A-11: (a) Bulk density and (b) bulk density percent change. The background density in our model (dark blue) is 2.25 g/cc.

Figure A-12: Time-lapse gravity change as a function of depth and radius from the injection well center. Profiles are at constant depth.

A-6 Deformation

Geodetic techniques measure displacements or displacement gradients at the earth’s surface. Such techniques are commonly used in the study of earthquakes or volcanoes, but they may also have applications in monitoring CO2 sequestration under certain conditions. In a stable tectonic environment, measured deformation over a sequestration site should only be the result of induced pressure changes at depth due to fluid injection. The magnitude of deformation resulting from a point source, also called a “nucleus of strain”, is known. Integrating this point solution over the region of pressure change will produce an arbitrarily complex surface deformation model. Such techniques have been used to explain land subsidence associated with oil production[17].
A-6.1 Deformation Model

In a mechanical sense, fluids only contribute to surface deformation through the pore pressure changes that they carry. That said, the more compressible the reservoir fluids and the larger the volume of reservoir fluid, the less pressure buildup will occur. The deformation model we use is to couple poroelastic theory with the so-called Mogi model [18]. The solution is given by

\[
\mathbf{u}(\mathbf{x}) = \frac{1}{2\pi} \int dV \frac{1 - 2\nu}{\mu} \alpha p(\xi) \frac{x_i - \xi_i}{|\mathbf{x} - \mathbf{\xi}|^3}
\]

where \( \mathbf{u} \) is the displacement vector at point of measurement \( \mathbf{x} \) on the free surface \( (x_3 = 0) \) and \( \xi \) is the spatial variable for the distribution of pressure change. Important to note is that the magnitude of deformation is linearly related to pressure change, inversely related to the rock modulus, and falls off inversely with distance squared.

An important difference between surface subsidence seen in producing oil fields and uplift expected from sequestration lies in the magnitude of expected deformation. While the fundamental equations are unchanged between the two cases, the effective modulus may be very different. This occurs mostly in poorly consolidated sandstones as increasing effective stress, by decreasing the pore pressure with a constant overburden, results in the compaction of the rock along the “virgin curve”[19] a process which changes the moduli of the rock. Decreasing the effective stress, however, causes the rock to unload on a different curve described by Eberhart-Phillips (Figure A-13), which is nearly elastic for small pressure perturbations.

![Figure A-13: Bowers’ virgin curve and Eberhart-Phillips unloading curve behavior.](image)

A rule of thumb is that the ratio of the compaction coefficient to rock compressibility is approximately ten to one, meaning that ten times as much subsidence is expected to be associated with a negative pore pressure change as would occur from a positive pressure change of the same magnitude. For subsidence, depending on the type of reservoir rocks present, either elastic compression or inelastic compaction may occur. Elastic compression normally occurs in very hard rocks like carbonates or upon reloading of sandstones.
A-6.2 Reservoir Scale Simulations

Figure A-15 shows the deformation and tilt results associated with the pressure changes given by our model (Figure A-14). One millimeter of displacement is well below the detectability threshold of modern instruments; instrument sensitivity is typically on the order of one centimeter of vertical resolution for continuous GPS and InSAR (interferometric synthetic aperture radar). GPS has the better sensitivity while InSAR is desirable because of the low cost of processing and the wide spatial coverage. Given this sensitivity, depending on the depth, rock type, and pressure changes detecting signals from sequestration may not be possible using either of these techniques. A more useful technique may be the use of tiltmeters as the one microradian predicted by our modeling represents a very detectable signal. Tiltmeters may have sensitivities as low as 0.1 microradian, below the peak expected signal from only one year of injection in our model.

**Figure A-14:** Pore Pressure change in MPa from initial pore pressure.

**Figure A-15:** Deformation results after 10 years of injection. (a) Vertical surface displacement and (b) surface tilt for a surface profile passing over the injection well.

Surface geodetic techniques, much like gravity, are low-resolution techniques. Source geometry is poorly constrained for inversion and as such it is likely that in any sequestration monitoring application would be focused more on mass balance and bulk storage of on CO₂ than on CO₂ migration and leak detection. Downhole tiltmeters have been suggested for measuring deformation at a more local level. Such instruments could
potentially detect deformation associated with hydrofracs in low porosity carbonates[1], as well as providing more focused data on pore pressure changes.

As the signal is strongly dependent on the depth, rock type, and pressure buildup, sites will need to be assessed on a case-by-case basis for usefulness of these geodetic techniques. In general, smaller systems with closed boundaries, as are commonly found in oil reservoirs, will lead to larger pressure changes and greater signals. Large open systems, like brine aquifers, may have virtually no pressure changes if permeability is sufficiently high. The presence of a large gas cap would also have a significant effect on pressure changes. Having a large volume of highly compressible gas would reduce any pressure increase resulting from injection.

A-7 Conclusions

While it would be very difficult to accurately predict the values of the physical parameters in a real formation either before or after CO₂ injection, accurately predicting time-lapse changes from a baseline survey is much easier. Most of the geophysical models used here are either empirical relations or only true for an idealized isotropic elastic material. Using models such as these provide only approximate solutions but give valuable insight into the behavior of these systems.

The results of the above discussions are summarized in Table A-1. Not surprisingly seismic, being the highest resolution technique, has the widest range of uses and is not limited by geologic setting except as previously noted. The SACS project at Sleipner has certainly confirmed the ability of seismic monitoring to track CO₂ in the subsurface. High resolution 3-D seismic is also one of the most expensive techniques to use, costing on the order of a million dollars per survey. While this cost is high, when compared to the expected costs sequestration it should not constitute a very significant expense[20].

These monitoring techniques also need not be used independently. LBNL conducted at study at the Lost Hills field in southern California during a CO₂ injection pilot study [21]. They used both high resolution crosswell seismic and electromagnetic monitoring to find compressional and shear velocities as well as conductivity. Using the combination of these methods they were able to separate pressure and saturation changes from R_{CO₂} effects. Using combination of techniques to constrain our models may prove necessary to reach our monitor goals for CO₂ sequestration.
Appendix B - Dynamic Diffusion Model for DARS Permeability Estimation

Wave propagation in a fluid-saturated porous medium results in complex interactions between the saturating fluid and the solid matrix. The presence of fluid in the pore space makes the elastic moduli frequency-dependent. The bulk modulus of a porous medium involves information about the flow properties of the medium. Because the micro-flow associated with acoustic wave does not involve mass transportation of the pore fluid, we call it dynamic flow to distinguish it from conventional flow. A dynamic diffusion model that relates the effective compressibility to the permeability is derived and applied to the interpretation of DARS experimental results.

In DARS (Differential Acoustic Resonance Spectroscopy), a standing wave inside the cavity provides a spatially varying but harmonic pressure field in the cavity. In a fluid-saturated porous medium that is subjected to this small-amplitude oscillatory pressure gradient, the pressure fluctuation will cause micro-scale fluid flow through the surface of the sample to release the differential pressure across the surface boundary. The net mass transport of the pore fluid is zero; therefore, this micro-scale flow behaves differently from conventional fluid flow. This dynamic flow phenomenon can be described as a quasi-static diffusion process. If the porous medium is homogeneous, the dynamic flow can be modeled by a 1D diffusion equation

\[
\frac{\partial^2 p}{\partial x^2} = \frac{1}{D} \frac{\partial p}{\partial x}, \quad (B-1)
\]

with diffusivity \( D \) given by \( D = k / \phi \eta \beta \). Here, \( \phi \) and \( k \) are porosity and permeability of the porous sample, respectively, \( p \) is the acoustic pressure in the fluid, \( \eta \) is the viscosity of the fluid, and \( \beta \) is the compressibility factor involving both the fluid and the solid matrix simultaneously. If acoustic pressure is harmonic in time, \( p(x,t) = p(x) e^{i\omega t} \), we can rewrite equation B-1 as

\[
\frac{\partial^2 p}{\partial x^2} - \frac{i \omega}{D} p = 0. \quad (B-2)
\]

A general solution of equation B-2 is

\[
p(x) = Ae^{\alpha x} + Be^{-\alpha x}. \quad (B-3)
\]

Here, \( \alpha = \sqrt{i \omega / D} \) and \( \omega \) is angular frequency.

In our particular case, the dynamic flows are in and out the sample at the two open ends when the exciting mode has longitudinal pressure variations; therefore, the pressure distribution inside the pore space is a superposition of two opposite pressure profiles, with boundary conditions \( p(L) = p_0 \) and \( p(-L) = p_0 \), respectively, when the sample is at the center of the cavity. Applying these two boundary conditions, we get the solution of the pressure field inside the porous sample,

\[
p(x) = \frac{e^{\alpha L}}{1 + e^{2\alpha L}} \left( e^{\alpha L} + e^{-\alpha L} \right) p_0. \quad (B-4)
\]
The effective compressibility of fluid-saturated porous materials under a periodic load can be expressed by the ratio of the net volumetric strain of the material to the stress applied on the sample. The net volume change of the sample consists of contributions from the solid matrix and the pore fluid. Therefore, the effective compressibility of the porous sample can be written as

$$
\kappa_e = -\frac{1}{V_s} \frac{(\Delta V_m + \Delta V_f)}{p_0},
$$

(B-5)

where $V_s$ is the bulk volume of the sample. $\Delta V_m$ is the volume change of the frame (the wet-frame in this case, because the sample is saturated), and $\Delta V_f$ is the volume of the extra amount of fluid flowing in and out the pore space; $p_0$ is the amplitude of pressure change. Here we assume the compressibility of the wet matrix is $\kappa_u$, hence, $\Delta V_m$ can be expressed as

$$
\Delta V_m = -\kappa_u V_s p_0.
$$

(B-6)

The parameter $\kappa_u$ is defined to be the undrained wet-frame compressibility for fluid-saturated porous materials. This parameter is also recognized as the reciprocal of the Gassmann wet-frame bulk modulus.

In a cylindrical porous sample with a jacketed side surface, diffusion happens only at the two open ends. The volume of the free-flowing fluid can be quantified as follows

$$
\Delta V_f = -\int \phi \kappa_f p(x) dV = -\pi a^2 \phi \kappa_f \int p(x) dx.
$$

(B-7)

Rewriting equation B-5 by substituting equations B-4, B-6 and B-7 into it, we get the final expression for the effective compressibility,

$$
\kappa_e = \kappa_u + \frac{\phi \kappa_f}{\alpha L} \frac{e^{2\alpha L} - 1}{e^{2\alpha L} + 1}.
$$

(B-8)

The effective bulk modulus is simply the inverse of the effective compressibility. The second term on the right hand side of equation B-8 is named as the dynamic flow component of compressibility. If $\kappa_e$ and $\kappa_u$ in equation B-8 are measured as described in Section 1.4, then we can solve equation B-8 for $\alpha$ alpha or permeability if other parameters are known.
Appendix C - Seismic Simulation Tools

Full-waveform seismic simulation is essential for developing new monitoring methods, studying survey design, testing processing schemes and algorithms, and for interpreting the imaging results. Moreover, it is important to incorporate the latest-known physics into the simulation methods. In our case, we need to include attenuation. Seismic attenuation has not been studied for subsurface monitoring. Changes in seismic attenuation (and velocity) associated with saturation changes should be observable as a monitoring signature. We have developed a suite of modeling tools, some with complex fluid-solid physics. The GCEP suite includes semi-analytic methods (R/T) and finite-difference (FD) modeling codes to simulate wave propagation in acoustic, elastic, visco-elastic, and poro-elastic media. In Table B-1 we list the suite of modeling tools developed for GCEP. Only a newly developed finite-difference method is presented in Appendix C for details.

Table C-1: The GCEP full-waveform seismic modeling suite includes visco-elastic and poro-elastic codes for the study of attenuation.
Appendix D - Optimized Variable-grid Finite Difference Method for Seismic Modeling

Finite Difference (FD) methods have historically dominated elastic wavefield modeling in geophysics because of their flexibility in representing complex models and their computational efficiency. Recently, variable-grid FD techniques have been developed to avoid spatial oversampling when applied to multi-scale structures[23] or large-scale structures with high velocity contrasts[24]. However, variable-grid FD methods developed based on the Taylor series approximation may suffer unacceptable dispersion.

We have developed an optimized fourth-order staggered-grid FD operator on a mesh with variable grid spacing based on the idea of the DRP (dispersion-relation-preserving) scheme[25]. The philosophy of the DRP method is to optimize the FD scheme coefficients by matching the effective wave number and the actual wave number over a particular wave number range. Comparison of the spectral properties between the optimized and the Taylor variable-grid FD schemes illustrates that the optimized scheme has less dispersion errors than the Taylor scheme[24] with the same stencil. We apply this technique for a solution of 2-D velocity-stress elastic wave equations and demonstrate the accuracy and efficacy of this method on some numerical examples.

C-1 Optimized Variable-grid FD Operator

The optimized variable-grid FD operator derived here is based on the idea of the DRP scheme proposed by [25]. To illustrate the problem, the 1-D velocity-pressure wave equations are considered (because spatial derivatives with respect to x, y and z are decoupled, the 1-D wave equation illustration will not lose generality):

\[
\begin{align*}
\rho \frac{\partial v}{\partial t} &= \frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial t} &= K \frac{\partial v}{\partial x},
\end{align*}
\]

where \(p\) and \(v\) are pressure and particle velocity, respectively; \(\rho\) is density and \(K\) is bulk modulus. Discretizing these equations by a staggered-grid FD mesh with variable grid spacing yields the scheme shown in Figure D-1.

![Figure D-1: (a) Schematic representation of unit cells and (b) the variable grid spacing.](image)

Suppose that the field variable \(g\) represents particle velocity \(v\) or pressure \(p\). The approximation of the first-order spatial derivative \(\partial g/\partial x\) by a fourth-order FD operator on a non-uniform grid of spacing \(dx\) is given by
where \( c_i \) are four coefficients to be determined. Spatial increments \( \Delta_i \) can be expressed in terms of the variable grid spacing \( dx \) (Figure D-2). After Fourier transform of equation D-2, the effective numerical wave number of the FD scheme can be calculated by

\[
k_e = -i(c_1 e^{ik\Delta_1} + c_2 e^{-ik\Delta_2} + c_3 e^{ik\Delta_3} + c_4 e^{-ik\Delta_4}).
\]  

(D-3)

For the optimized FD scheme, the coefficients \( c_i \) in equation D-3 are chosen so that the effective wave number \( k_e \) is close to the actual wave number \( k \) for a wide range of wave numbers. The coefficients \( c_i \) are determined by imposing the condition that equation D-3 is accurate to the third-order of \( \Delta_i \) through Taylor series expansion:

\[
\begin{align*}
    c_1 + c_2 + c_3 + c_4 &= 0 \\
    c_1\Delta_1 - c_2\Delta_2 + c_3\Delta_3 - c_4\Delta_4 &= 1 \\
    c_1\Delta_1^2 + c_2\Delta_2^2 + c_3\Delta_3^2 + c_4\Delta_4^2 &= 0
\end{align*}
\]

(D-4)

Figure D-2: Grid nodes with variable spacing. \( \Delta_i \) (i = 1, 4) are used to calculate the FD operator centered between (a) the nodes \( i \) and \( i+1 \) and (b) that centered at the node \( i \).

This leaves one of the coefficients, e.g., \( c_1 \), as a free parameter. This parameter is then chosen to minimize the integrated error \( E \) defined as

\[
E = \lambda \int_{0}^{\eta} (k - \text{Re}(k_e))^2 dk + (1 - \lambda) \int_{0}^{\eta} (\text{Im}(k_e))^2 dk,
\]

(D-5)

where \( \eta \) is a predetermined number that gives the optimized range of wave numbers. The weighting coefficient \( \lambda \), is used to balance the L2 norm of the truncation errors of the approximation of the real and imaginary parts of the effective numerical wave number to the actual wave number. The necessary condition used to minimize \( E \) is

\[
\frac{\partial E}{\partial c_1} = 0
\]

(D-6)
From equation D-6, we can get $c_1$ analytically. Then $c_2$, $c_3$, and $c_4$ can be obtained from equation D-4.

We compare the spectral properties of this optimized variable-grid FD operator and the same order Taylor variable-grid FD operator[24] for different variable-grid meshes. Figure D-3 shows the relation between $\text{Re}(k_e)dx_{\text{min}}$ and $kdx_{\text{min}}$ of both schemes for the stencil in Figure D-2(a) with the spacing ratio($r$) between the coarse grid and the fine grid of 1, 3 and 6. The closer the curves are to the exact relation $\text{Re}(k_e)dx_{\text{min}}=kdx_{\text{min}}$, the smaller the dispersion. Figure D-3 demonstrates that the optimized FD scheme has less dispersion errors than the FD scheme based on Taylor series expansion with the same mesh. In fact, the spectral resolution properties of the optimized FD scheme with the grid spacing ratio of 6 is much better than that of the Taylor variable-grid FD scheme with the same stencil and is even close to that of the Taylor regular grid FD scheme (spacing ratio of 1). This means that the optimized variable-grid FD scheme with large spacing ratios between two grid domains can give the same accurate results as the Taylor variable-grid FD scheme with small spacing ratios. Therefore, the optimized variable-grid FD technique can lead to either more accurate results with the same number of grid cells or more efficient calculations with fewer cells compared to the Taylor variable-grid FD method.

![Figure D-3: $\text{Re}(k_e)dx_{\text{min}}$ versus $kdx_{\text{min}}$ of the optimized and Taylor variable-grid FD schemes for the stencil in Figure D-2(a). $r$ represents the spacing ratio of the coarse grid to the fine grid. The exact relation is $\text{Re}(k_e)dx_{\text{min}}=kdx_{\text{min}}$.](image)

D-2 Applications of the optimized variable-grid FD operator

To test the accuracy and the efficiency of the proposed optimized variable-grid FD scheme, we implement it for a solution of 2-D velocity-stress elastic wave equations. The following are two numerical examples.
D-2.1 Homogeneous model

The homogeneous model with physical properties: $V_p = 2300$ m/s, $V_s = 1100$ m/s and $\rho = 2100$ kg/m$^3$, is shown in Figure D-4. We use this model to test the accuracy of the optimized variable-grid FD scheme when the grid spacing increases abruptly from 2 m to 6 m with a ratio of 3 in both x and z directions. The dashed line in Figure D-4 represents the boundary between the small grid spacing domain and the large grid spacing domain. A vertical dipole source (S), located at (450 m, 450 m), has a Ricker pulse with central frequency of 20 Hz. Receivers R$_1$ and R$_2$ are placed on the left- and the right-hand sides of the domain boundary, respectively.

![Figure D-4: Homogeneous model.](image)

Figures D-5 and D-6 present the comparison of the horizontal and vertical displacements calculated by the optimized variable-grid FD scheme with those obtained from the analytical solutions[26] and the Taylor regular-grid FD scheme with a constant grid spacing of 2 m at receiver R$_1$ and R$_2$, respectively. The agreement among the three solutions is very good. Note that the FD schemes with variable- and regular-grid spacing give essentially identical results indicating both the reflection and the transmission artifacts from the domain boundary of the variable-grid FD scheme are negligible.
Figure D-5: Comparison of optimized variable-grid FD method (OVGFD) with the analytic solutions and the Taylor regular-grid FD scheme (TRGFD) at receiver $R_1$: (a) horizontal displacement; (b) vertical displacement.

Figure D-6: Comparison of optimized variable-grid FD method (OVGFD) with the analytic solutions and the Taylor regular-grid FD scheme (TRGFD) at receiver $R_2$: (a) horizontal displacement; (b) vertical displacement.
D-2.1 Thin-layer model

The accuracy of the optimized variable-grid FD technique has been verified for a homogeneous region. Now we apply it for modeling wave propagation in a thin-layer model (Figure D-7). The size of the model is 6000 m × 2000 m. A water-filled thin layer with 1 m thickness is embedded in the model. The purpose of the modeling is to see if the effects of the thin fluid-filled layer can be observed in the seismograms. Table D-1 provides the physical properties of the model. The source, centered in y direction and located 600 m below the top, is a 20 Hz Ricker wavelet in pressure. Receivers are on both sides of the source and on the same level as the source. To resolve the thin layer with 1 m thickness, a very fine mesh (at least 0.2 m) is required. An equally spaced mesh with sufficient resolution to describe the thin layer in this large-size model requires too much memory for most computers. For the optimized variable-grid FD method, the vertical grid spacing smoothly increases from 0.2 m to 5.4 m with a 7.2 m wide transition region. The horizontal grid spacing is 5.4 m throughout the grid. The total grid size for this variable-grid mesh is $N_X \times N_Z = 1225 \times 517$ which is only 4.08 percent of the total grid size for the regular-grid mesh with constant $dx = 0.2$ m in the entire model ($N_X \times N_Z = 30000 \times 517$); therefore, the variable-grid FD scheme saves over 95 percent computer memory for this model compared to the regular-grid FD scheme. Consequently, the computation time is reduced by more than 24 times.

<table>
<thead>
<tr>
<th>No.</th>
<th>$V_p$(m/s)</th>
<th>$V_s$(m/s)</th>
<th>$\rho$(kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3000</td>
<td>1700</td>
<td>2300</td>
</tr>
<tr>
<td>2</td>
<td>1500</td>
<td>0</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Figure D-7**: Thin-layer model.

Figure D-8 shows the wavefield snapshots of horizontal and vertical particle velocities. Reflections and transmissions from the thin water-filled layer can be seen. No artifacts are generated by smoothly varying spacing in the vicinity of the thin layer. Seismograms of the horizontal and vertical particle velocity components are shown in Figure D-9. We can clearly see the reflected P- and S-waves from the thin layer in
addition to the direct P-wave. This modeling study demonstrates that we need to include the effects of thin fluid-filled layers into the field data analysis.

Figure D-8: Snapshots of horizontal particle velocity ($V_x$) and vertical particle velocity ($V_z$) components of the thin-layer model. Reflections and transmissions from the thin layer can be seen.

Figure D-9: Seismograms of horizontal particle velocity ($V_x$) and vertical particle velocity ($V_z$) components of the thin-layer model. Reflected $P$- and $S$-waves from the thin water-filled layer are observed in addition to the direct $P$-wave.

In conclusion, this optimized variable-grid FD scheme has less dispersion errors than the Taylor variable-grid FD scheme with the same stencil. Numerical examples demonstrate that the proposed technique can efficiently and accurately simulate wave propagation in large models with physically small features.
Appendix E - Seismic Monitoring of Coal Bed Methane Production: A Case Study from Powder River Basin, Wyoming

CO₂ storage in coal beds is an important geological sequestration option. This is a cost-effective storage if associated with methane production, because injected CO₂ may enhance the methane production. Depending on the coal rank, coal generally has high CO₂ adsorbing capacity. Coal has global and vast geographical distribution and is often close to CO₂ emission sources. The field study presented here is an example of seismic monitoring for coal bed methane (CBM) production and there were no CO₂ storage activities involved yet. This field test showed that the velocity changes caused by gas in coal are detectable by the seismic method. The results of this monitoring experiment should provide useful information for future CO₂ sequestration project.

E-1 Objective

This CBM project was carried out in Powder River Basin, Wyoming. The primary goal of this project is to pump out the water in coal beds and increase CBM production. This dewatering process will reduce the pore pressure and increase gas saturation. Therefore, the depressurization can enhance the CBM production[32,33]. To monitor the dewatering process, three crosswell seismic surveys were acquired in different times. Higher gas saturation lowers P-wave velocity. Lower pore pressure (or higher effective differential pressure) will increase both P-wave and S-wave velocities. The objective of monitoring is to detect the time-variant distributions of the seismic velocities, and then infer the spatial distribution of gas between wells. The results might be helpful in identifying both the source of water produced and the spatial efficiency of the dewatering process.

E-2 Data Acquisition

Two observation wells, spanning 150ft and straddling a production well were used for the crosswell surveys (see Figure E-1). The baseline or reference survey was acquired in December 2002, shortly after the production well was completed and before dewatering began. Two time-lapse monitoring surveys were run in July 2003 and June 2004, respectively. Each of the surveys covered about 900ft to the total depth of the wells at about 1400ft. Shots were fired from a down-hole piezoelectric source every 1.25ft in the source well. The seismic waves propagated between wells were picked up by an array of hydrophones positioned inside the receiver well. There were 182 source positions and 116 receiver positions.

E-3 Data Processing

Let us first intuitively inspect the data quality and some of simple features in the data through zero-offset gathers (source and receiver are at the same depth levels). Figure E-2 shows three zero-offset gathers extracted from three surveys. We only expect that seismic events with the coal bed have relatively large change between time-lapse surveys. We place red markers on first arrivals within and outside the coal bed in Survey 1 as shown in Figure E-2. We then projected same markers onto Surveys 2 and 3, respectively. As displayed in the resultant images, the arrival time outside the coal bed changes little from survey to survey, which means the repeatability of the data acquisition is good. The travel time within the coal bed has visible shift from survey to survey. The

72
travel time of Survey 2 is longer than Survey 1. However, the travel time of Survey 3 is shorter than Survey 2 and close to Survey 1. We will try to explain this observation later.

**Figure E-1**: Crosswell seismic survey geometry

*P*-wave first arrivals are picked and used for velocity tomography. Figure E-3 is the velocity tomogram for baseline survey. Gamma ray logs from the source and receiver wells are plotted alongside the tomogram to show the geologic structures. We also reconstruct the velocity tomograms for Survey 2 and Survey 3 shown in Figure E-4. To compare *P*-wave velocity changes due to dewatering, we compute the differences between different tomograms as shown in Figure E-5. The first difference tomogram ($\Delta V_{p1}$) is computed from tomograms for the baseline survey of December 2002 and first repeat survey of July 2003 (after about 8 months of dewatering). The second difference tomogram ($\Delta V_{p2}$) is computed from the tomograms for the baseline survey of July 2002 and second repeat survey of June 2004 (after about 19 months of continuous dewatering). The third difference tomogram ($\Delta V_{p3}$) is computed from two repeat tomograms.

**E-4 Data Interpretation**

Four distinct geologic units can be identified from gamma ray logs and baseline tomogram. They include shaly-sand, sandy-shale, coal, and sandstone. Of interest to us is the low-velocity, biogenic and low-rank Big George coal zone at a depth of 1150ft to 1240ft. The top of the coal aquifer is confined by the low permeability sandy-shale.
Figure E-2: A display of zero-offset sections for the three data sets showing evidence of production-induced changes within the coal reservoir and repeatability of events outside the reservoir.

Figure E-3: Baseline tomogram with gamma ray logs. Insets are tomogram-derived velocity logs.
The first difference tomogram $\Delta V_{P1}$ shows a 4-5% reduction in $V_P$ within the coal zone, perhaps due to partial gas saturation and/or methane desorption. We can see from $\Delta V_{P2}$ that $P$-wave velocity within the coal zone has little change between Survey 3 and Survey 1; that means that the velocity increases 4-5% from Survey 2 to Survey 3 (see the difference tomogram $\Delta V_{P3}$ in Figure E-5). Figure E-6 gives a possible explanation on interesting up-down changes in velocities. The $P$-wave velocity changes are caused by a mixing effect of gas saturation and pore-pressure changes. At the beginning of the dewatering, the gas saturation increasing is dominant and reduces overall velocity. At later time, the dewatering mainly increases differential pressure. Differential pressure is defined as confining pressure minus pore pressure. Higher differential pressure causes the closing presumably layer cavities or air-filled cracks [34] in the coal, which makes the wave travel faster. This is may explain the $P$-wave velocity increase from Survey 2 to Survey 3.

Figure E-6 is created based on the laboratory $P$-wave data on a Permian coal sample [35] and Gasmann’s equation [36]. We conditioned the data to reflect the prevailing georeservoir conditions at the PRB and the observed baseline coal velocities. The sample has porosity of 2.9% and density of 1.35g/cc. We also use appropriate fluid properties and coal physics relationships for $P$-wave velocity, effective fluid modulus and density of a fluid-saturated rock [37].

This field test showed that the velocity changes caused by gas in coal are detectable by the seismic method. In order to separate the saturation and pressure effects on velocities, we need to use $S$-wave information. In this study, we only used $P$-wave first arrivals, because the $S$-wave signal is too poor to pick. The Western Resources Project sponsored the data acquisition for this study. GeoTomo provided certain software support.

Figure E-4: Baseline and repeat tomograms (from left: Survey 1, Survey 2 and Survey 3).
Figure E-5: Measured difference tomograms from the time-lapse surveys. From left is the 1st difference tomogram, $\Delta V_{p1}$ (Survey2 minus Survey 1); 2nd difference tomogram, $\Delta V_{p2}$ (Survey 3 minus Survey 1) and 3rd difference tomogram, $\Delta V_{p3}$ (Survey 3 minus Survey 2). Arrow shows the location of the production well.

Figure E-6: $P$-wave velocity in saturated coal as a function of differential pressure.
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