Adaptive Multiscale Method for Nonlinear Flow and Transport in Porous Media

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Introduction

Scale difficulty in modeling underground CO_2 sequestration

- Highly heterogeneous underground porous media requires fine-scale description (e.g., $10^6 \sim 10^8$ cells)
- Fine-scale geophysical model requires large-scale computational system (very difficult and slow to solve)

Multiscale methods promising to resolve this scale issue

- Solve global system in coarse scale
- Reconstruct fine-scale information locally
- Coarse-scale global solver and fine-scale local solver are intertwined to improve quality of both solvers

Governing Equations

Mass balance equations for black-oil model:

$$\frac{\partial(\phi b_l S_l)}{\partial t} + \nabla \cdot (b_l u_l) = q_l,$$

$$u_l = -\lambda_l \nabla p, \quad \lambda_l = \frac{k k_{rl}}{\mu}, u_T = \sum_l u_l$$

Sequential implicit time discretization (linearized form):

$$C\frac{p^{\nu+1} - p^{\nu}}{\Delta t} - \sum_l \alpha_l^{\nu} \nabla \cdot (b_l^{\nu} \lambda_l^{\nu} \nabla p^{\nu+1}) = R,$$

$$\frac{\phi^{\nu+1} b_l^{\nu+1} S_l^{\nu+1} - \phi^n b_l^n S_l^n}{\Delta t} = \nabla \cdot \left\{ b_l^{\nu+1} f_l^{\nu+1} u_T^{\nu+1} \right\} - q_1,$$

An Algebraic Multiscale Framework

$$\mathbf{A}_f \mathbf{p}_f = \mathbf{r}_f$$

$$\mathcal{P} \downarrow \mathbf{p}_f = \mathcal{P}\mathbf{p}_c$$

$$\mathbf{A}_f \mathcal{P} \mathbf{p}_c = \mathbf{r}_f$$

$$\mathcal{R} \downarrow$$

$$\mathcal{R}\mathbf{A}_f \mathcal{P} \mathbf{p}_c = \mathcal{R}\mathbf{r}_f$$

$$\mathbf{A}_c = \mathcal{R}\mathbf{A}_f\mathcal{P} \downarrow \mathbf{r}_c = \mathcal{R}\mathbf{r}_f$$

$$\mathbf{A}_c \mathbf{p}_c = \mathbf{r}_c$$
$$\mathbf{p}_f = \mathcal{P}\mathbf{p}_c$$

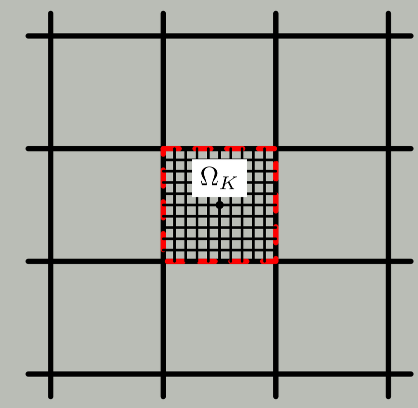
$\mathcal{R}$: Multiscale Restriction Operator

- Map fine-scale equations to coarse-scale equations
- Finite-volume type restriction operator

$$\mathcal{R}_{K,m} = \begin{cases} 1 \text{ if } \Omega_m \subset \Omega_K \\ 0 \text{ otherwise} \end{cases}$$
$$(K = 1, \ldots, N_c; m = 1, \ldots, N_f).$$

- Finite-element type restriction operator

$$\mathcal{R} = \mathcal{P}^T$$



$\mathcal{P}$: Multiscale Prolongation Operator (1) For Pressure

- Map coarse-scale pressure to fine-scale pressure

$$\mathbf{p}_f = \mathcal{P}\mathbf{p}_c$$

- Dual pressure basis function

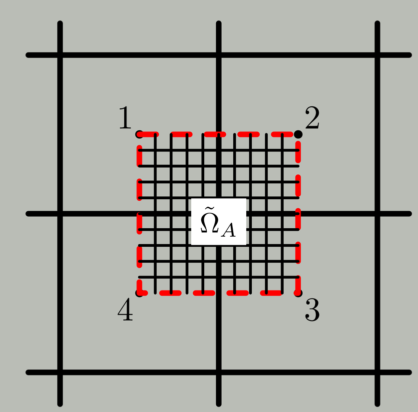
$$\sum_{l=1}^{n_p} \alpha_l^{\nu} \nabla \cdot \left(b_l^{\nu} \lambda_l^{\nu} \nabla \phi_A^i \right) = 0 \quad \text{in } \tilde{\Omega}_A,$$

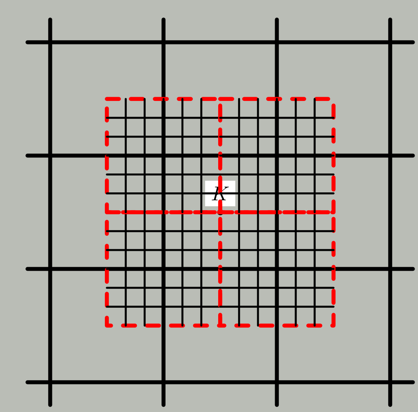
$$\sum_{l=1}^{n_p} \alpha_l^{\nu} \frac{\partial}{\partial x_t} \left(b_l^{\nu} \lambda_l^{\nu} \frac{\partial \phi_A^i}{\partial x_t} \right) = 0 \quad \text{on } \partial\tilde{\Omega}_A,$$

$$\phi_A^i(\mathbf{x}_j) = \delta_{ij}.$$

- Assemble prolongation operator from local basis functions

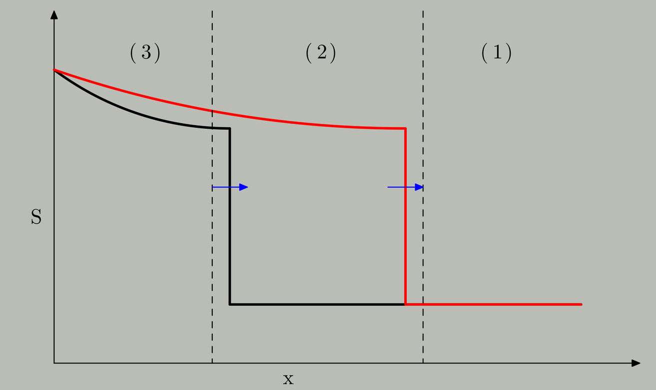
$$\Phi_K = \sum_{\tilde{\Omega}_A \in supp(\Phi_K)} \phi_A^i$$
$$\mathcal{P}_{m,K} = \Phi_K(\mathbf{x}_m)$$





$\mathcal{P}$: Multiscale Prolongation Operator (2) For Saturation

- No general mapping from coarse-scale saturation to fine-scale saturation
- Different local solution strategies are adaptively used to obtain fine-scale saturation
- Computational domain can be divided into three regions:



1. ahead-of-front region
2. front region
3. behind-front region

Solution Strategies for Transport Equation

Update total velocity

- Full construction: solve Neumann problems locally
- Approximate updating: Location weighted linear interpolation

$$\Delta u_x^h = \frac{(x - x_0)\Delta U_x^H(x_0) + (x_1 - x)\Delta U_x^H(x_1)}{x_1 - x_0}$$

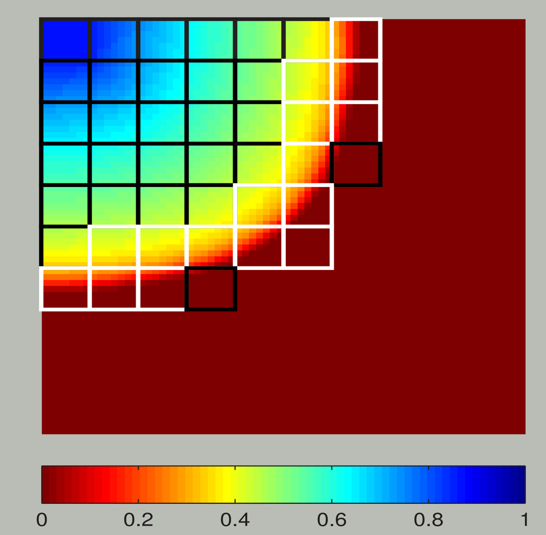
Update fine-scale saturation

- Full construction: solve original transport equations
- Approximate updating: History weighted linear interpolation

$$\Delta S_i^h = \xi_K^i \Delta S_K^H$$

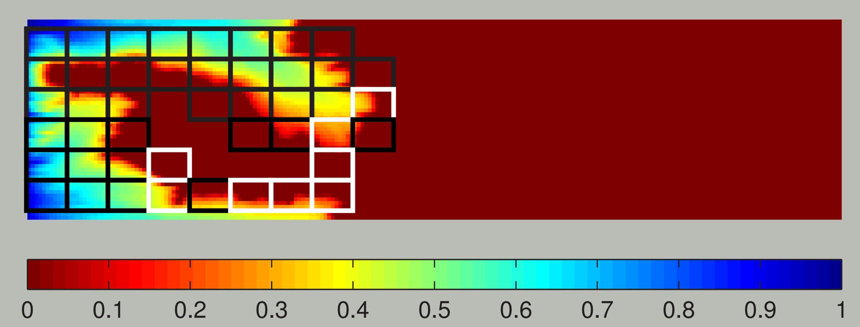
where $\xi_K^i = \frac{\delta S_i^h}{\delta S_K^H}$ for $\Omega_i^h \in \Omega_K^H$ from history.

Numerical Examples



Homogeneous case: Error and Adaptivity

e_p	2.09e-5
e_s	5.05e-5
basis(%)	3.88
velocity(%)	10.67
transport(%)	15.22



SPE 10 top layer: Error and Adaptivity

e_p	7.23e-5
e_s	3.55e-5
basis(%)	3.76
velocity(%)	4.20
transport(%)	15.55

Summary

- Developed a general algebraic multiscale framework that can incorporate different physics for multiscale computation easily.
- Proposed an adaptive multiscale formulation that is accurate, efficient and robust for general blackoil problems.
- The adaptive multiscale method will be very promising to model large-scale CO_2 sequestration problem.

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