Adaptive Multiscale Method for Nonlinear Flow and Transport in Porous Media

Hui Zhou  Hamdi Tchelepi
Department of Energy Resources Engineering, Stanford University

Introduction

Scale difficulty in modeling underground CO₂ sequestration
- Highly heterogeneous underground porous media requires fine-scale description (e.g., $10^6$ $-$ $10^8$ cells)
- Fine-scale geophysical model requires large-scale computational system (very difficult and slow to solve)

Multiscale methods promising to resolve this scale issue
- Solve global system in coarse scale
- Reconstruct fine-scale information locally
- Coarse-scale global solver and fine-scale local solver are intertwined to improve quality of both solvers

Governing Equations

Mass balance equations for black-oil model:

$$\frac{\partial (\phi b_i s_i)}{\partial t} + \nabla \cdot (b_i u_i) = q_i, \quad u_i = -\lambda_i \nabla p_i, \quad \lambda_i = \frac{kk_{li}}{\mu}, \quad \sum u_i$$

Sequential implicit time discretization (linearized form):

$$\frac{C^{p+1} - p^0}{\Delta t} - \sum_{i} \alpha_i^n \nabla \cdot (b_i^n \lambda_i^n \nabla p^{p+1}) = R_i \quad \phi_i^{p+1} b_i^{p+1} s_i^{p+1} - \phi_i^n b_i^n s_i^n = \nabla \cdot \left( b_i^{p+1} u_i^{p+1} \right) - q_i,$$

$\phi_i^{p+1}$: Multiscale Restriction Operator

$$R_{K,m} = \begin{cases} 1 & \text{if } \Omega_m \subset \Omega_K \\ 0 & \text{otherwise} \end{cases} \quad (K = 1, \ldots, N_k; m = 1, \ldots, N_l).$$

Finite-volume type restriction operator

Finite-element type restriction operator

Assemble prolongation operator from local basis functions

$\phi_i^A$:

$$\Phi_K = \sum_{\Omega_m \supset \text{supp}(\phi_i^A)} \phi_i^A$$

$\mathcal{P}_{m,K} = \Phi_K(x_m)$

$\mathcal{R}$:

$\mathcal{P}$:

Solution Strategies for Transport Equation

Update total velocity
- Full construction: solve Neumann problems locally
- Approximate updating: Location weighted linear interpolation

Update fine-scale saturation
- Full construction: solve original transport equations
- Approximate updating: History weighted linear interpolation

Numerical Examples

Homogeneous case: Error and Adaptivity

| $\varepsilon_p$ | 2.09e-5 |
| $\varepsilon_k$ | 5.05e-5 |
| basis(%) | 3.66 |
| velocity(%) | 10.67 |
| transport(%) | 15.22 |

SPE 10 top layer: Error and Adaptivity

| $\varepsilon_p$ | 7.23e-5 |
| $\varepsilon_k$ | 3.55e-5 |
| basis(%) | 3.76 |
| velocity(%) | 4.20 |
| transport(%) | 15.55 |

Summary

- Developed a general algebraic multiscale framework that can incorporate different physics for multiscale computation easily.
- Proposed an adaptive multiscale formulation that is accurate, efficient and robust for general blackoil problems.
- The adaptive multiscale method will be very promising to model large-scale CO₂ sequestration problem.

October 1-3, 2008  GCEP Research Symposium  {huizhou, tchelepi}@stanford.edu