Algebraic Multiscale Formulation for Compressible Multiphase Flow in Heterogeneous Porous Media

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Algebraic Multiscale Method

Background and Introduction

Problem:
Accurate modeling of flow and transport in large-scale highly detailed geologic models is a necessary requirement for making reliable predictions of CO2 sequestration processes.

Objective:
Solve large-scale (10^8 cells) highly heterogeneous flow and transport problems efficiently.

Multiscale – the Basic Idea:
The properties of natural porous media involve a hierarchy of complex scales and spatial correlation structures. As a result, flow and transport in natural geologic formations is a multiscale problem.

Multiscale formulations are designed to solve the fine-scale problem by building and solving special coarse-scale systems that allow for local reconstruction of the fine-scale information.

The power of existing multiscale formulations has been demonstrated for elliptic (e.g., incompressible) flow problems only. We developed a scalable and extendible algebraic multiscale framework that handles compressible multiphase flow in a natural way.

Algebraic Multiscale Method

Assume the linearized fine-scale equations can be cast into

$$\mathbf{A}_f \mathbf{p}_f = \mathbf{r}_f .$$

Construct two multiscale operators, namely, prolongation and restriction,

$$\mathbf{P}, \mathbf{R}.$$

The coarse scale system can be constructed as

$$[\mathbf{RAP}]\mathbf{p}_c = \mathbf{R}\mathbf{r}_f \Rightarrow \mathbf{A}_c \mathbf{p}_c = \mathbf{r}_c .$$

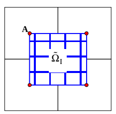
Solve this (small) coarse scale system. The fine scale is reconstructed by

$$\mathbf{p}_f = \mathbf{P}\mathbf{p}_c .$$

Multiscale Prolongation Operator

The prolongation operator is assembled from local basis functions

$$\left.\begin{aligned} &\nabla\cdot\left(\lambda\nabla\phi_A^{(I)}\right)=0 \quad \text{in } \tilde{\Omega}_I \\ &\frac{\partial}{\partial x_t}\left(\lambda\frac{\partial\phi_A^{(I)}}{\partial x_j}\right)_t = 0 \quad \text{on } \partial\tilde{\Omega}_I \\ &\phi_A^{(I)}(x_B)=\delta_{AB} \end{aligned}\right\}$$



$$\left\{\begin{aligned} &\phi_A=\sum_{I=1}^{n_c}\phi_A^{(I)} \\ &[\mathbf{P}]_{a,A}=\phi_A(x_a) \end{aligned}\right.$$

"A" denotes coarse node;
"a" denotes fine node

Formulation of Compressible Flow

Algebraic Form of the Restriction Operator

The form of restriction operator is based on coarse scale numerical schemes. For the finite volume method:

$$[R]_{A,a} = \begin{cases} 1 & \text{if } \Omega_a \subset \bar{\Omega}_A \\ 0 & \text{otherwise} \end{cases} \qquad (A=1,\dots,n_c;\ a=1,\dots,n_f)$$

For the Galerkin finite element method

$$\mathbf{R}=\mathbf{P}^T \Rightarrow [\mathbf{R}]_{A,a}=\phi_A(x_a)$$

This algebraic construction recovers existing multiscale methods for elliptic problems directly.

More importantly, the algebraic multiscale method offers great flexibility.

Algebraic Multiscale Formulation for Compressible flow

The general flow equation of compressible system is

$$\nabla\cdot(\mathbf{k}\cdot\lambda_t\nabla p)=\phi_0 c_t\frac{\partial p}{\partial t}+q$$

Assume the discrete form is

$$\mathbf{T}_f\mathbf{p}_f=\mathbf{C}_f\mathbf{p}_f+\mathbf{r}_f$$

where $\mathbf{T}_f$ is the flow part and $\mathbf{C}_f$ is the accumulation part that relates to compressibility.

The coarse scale system for compressible flow is

$$\left[\mathbf{R}\left(\mathbf{T}_f-\mathbf{C}_f\right)\mathbf{AP}\right]\mathbf{p}_c=\mathbf{R}\mathbf{r}_f \Rightarrow \left(\mathbf{T}_c-\mathbf{C}_c\right)\mathbf{p}_c=\mathbf{r}_c .$$

where $\mathbf{T}_f$ is the coarse scale flow part and $\mathbf{C}_f$ is the coarse scale accumulation part. The operators are defined the same as before.

Advantages of the Algebraic Multiscale Method

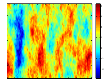
- Readily extendible to unstructured grid
- Easy to include complicated physics
- Allow for incorporating a multiscale formulation into existing reservoir simulators

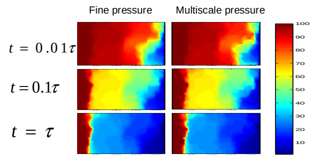
Numerical Results

Multiscale Solution for Flow Equation

Permeability field: top layer of SPE 10 comparative case

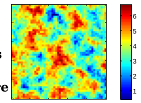
Study ideal gas flow in this highly heterogeneous reservoir

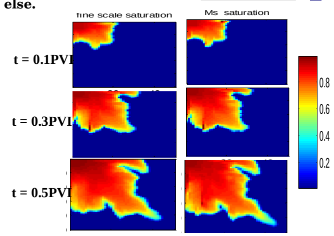




Multiscale Solution for Coupled Flow and Transport Equations

Both flow and transport equations are solved in multiscale method. This is an ongoing work and has not been done in anywhere else.





Future Work:

- Multiscale method for coupled flow and transport
- Multiscale method for flow with strong capillarity and gravity effects

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