Adaptive Multiscale Method for Flow and Transport in Porous Media

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Introduction

Problem:

Accurate modeling of flow and transport in large-scale highly detailed geologic models is a necessary requirement for making reliable predictions of CO_2 sequestration processes. However, it is very CPU costly to solve large-scale (10^8 cells) highly heterogeneous flow and transport problems using conventional fine-scale. On the other hand, the properties of natural porous media involve a hierarchy of complex scales and spatial correlation structures. As a result, flow and transport in natural geologic formations is a multiscale problem.

Background:

The flow and transport problems in natural porous media are very suitable to be solved using the multiscale method. Multiscale methods are designed to solve the fine-scale problem by building and solving special coarse-scale systems that allow for local reconstruction of the fine-scale information. Thus, both efficiency and accuracy are achieved. There has been extensive research on different multiscale methods. Among them, the multiscale finite volume method (MsFVM) is of the greatest interest due to its flexibility and efficiency. Our study is also within the framework of MsFVM.

Objectives:

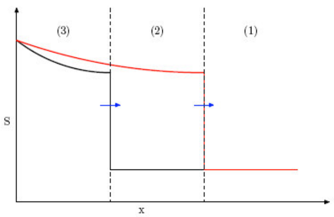
Previous research on multiscale method has greatly improved the efficiency of solving the flow problem (presssure part). However, the transport problem (saturation part) has not been solved in a multiscale approach mainly due to the difficulty of upscaling the hyperbolic transport equation. We propose an efficient and adaptive multiscale approach to solve the transport problems. By this approach, greater efficiency can be achieved over the previous multiscale method while maintaining the fine-scale accuracy.

Adaptive Multiscale Method for Transport

Solving transport in original MsFVM

1. Update fine-scale total velocity
   - To ensure the velocity field is locally conservative, fine-scale pressure need to be re-computed locally with flux boundary condition from flow problem
2. Solve fine-scale saturation
   - Of the same computational complexity as in conventional fine-scale simulator

Observations in typical saturation profile



1) ahead-front region
   - Saturation no changed

2) front region
   - Large changes in both total velocity and saturation

3) behind-front region
   - Small change in total velocity
   - Small or large change in saturation

Adaptive multiscale method for transport

1. Solve coarse scale velocity and saturation
2. Determine which saturation region each coarse block belongs to according to the coarse scale properties and criterion
3. Update fine-scale velocity and saturation adaptively
   - Region 1: no need to update saturation
   - Region 2: fine-scale computation of velocity and saturation as in original MsFVM
   - Region 3: interpolating velocity, interpolating or fine-scale computing saturation depending on whether the saturation is stabilized

Coarse scale transport equation

Fine-scale transport

$$A_i^h(S^h) = \sum_{i \in \aleph_i} f_{ij} u_{ij} - \beta_i (S_i^{h,n+1} - S_i^{h,n}) = r_i^h$$

Sum of fine-scale equations

$$A_K^H = \sum_{i \in \Omega_K} A_k^h = r_K^H$$

Coarse-scale transport

$$= \sum_{J \in \aleph_K} F_{Jk}^H U_{Jk}^H - \beta_I (S_i^{H,n+1} - S_i^{H,n})$$

$$S_K^H = \frac{1}{V_K} \sum_{i \in \Omega_K} v_i S_i^h$$

$$U_{JK}^H = \sum_{j,k \in \partial \Omega_{JK}^H} u_{jk}^h$$

$$F_{JK}^H = \frac{1}{U_{JK}^H} \sum_{j,k \in \partial \Omega_{JK}^H} f_{jk}^h u_{jk}^h$$

Interpolation of fine-scale velocity and saturation

1. Total velocity

$$\Delta u^h(\mathbf{x}) = \frac{\mathbf{x} - \mathbf{x}_0}{|\mathbf{x}_1 - \mathbf{x}_0|} \otimes \Delta U^H(\mathbf{x_0}) + \frac{\mathbf{x} - \mathbf{x}_1}{|\mathbf{x}_1 - \mathbf{x}_0|} \otimes \Delta U^H(\mathbf{x_1})$$

where $\mathbf{u} \otimes \mathbf{v} \equiv (u_1 v_1, u_2 v_2)$ in 2-D.

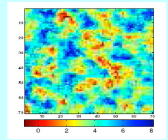
2. Saturation

$$\Delta S_i^h = \xi_K^i \Delta S_K^H$$

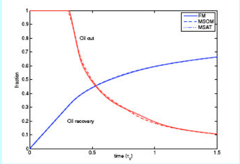
where $\xi_K^i = \frac{\delta S_i^h}{\delta S_K^H}$ for $\Omega_i^h \in \Omega_K^H$ from previous time step.

Numerical Results

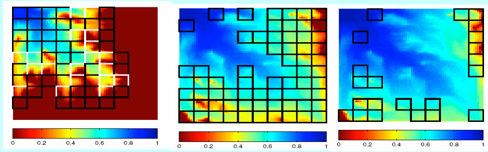
Two-phase flow in heterogeneous field



Log-normal perm.



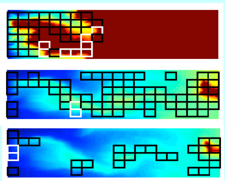
Oil recovery and production ratio
(FM – fine method, MSOM – original MS, MSAT – adaptive MS)



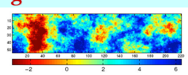
t = 0.2 PVI t = 0.8 PVI t = 1.5 PVI

White box: fine-scale computation of both sat. and vel.
Black box: fine-scale computation of sat. only
No box: no fine-scale computaton

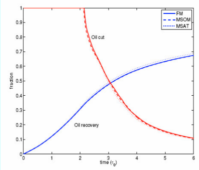
Two-phase flow in highly heterogeneous field



$t = \tau_0$
$t = 4\tau_0$
$t = 6\tau_0$



SPE10 top layer



CPU performance of adaptive multiscale simulator

number of cells	97,200	995,328
fine-scale CPU (min)	25.6	1618.7
original MS CPU (min)	13.3	281
adaptive MS CPU (min)	10.1	133

Summary

1. Developed the first adaptive multiscale formulation for both flow and transport problem
2. Demonstrated the accuracy and efficiency in highly heterogeneous permeability field with physics including gravity and compressibility

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