**Translational Lumped Parameter Model for HTSC - PM Interaction for Flywheel Bearing Applications**

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**Introduction**

High temperature superconducting bearings are a critical pathway to reduce losses for flywheel energy storage devices. Currently, flywheels which use active magnetic bearings have loss rates of 2 – 5% energy stored per hour. With superconducting bearings, these loss rates can be reduced to below 0.1% per hour [1], which enables flywheels to efficiently store energy on a diurnal scale.

Superconducting bearings consist of permanent magnet material that stably levitates over an array of bulk high temperature superconductors (HTSCs). Understanding the translational and vertical stiffness of such bearing designs is critical to flywheel rotors and performance. The University of Texas at Austin is developing lumped parameter models to describe the magnetic and electro-magnetic behavior between a levitated permanent magnet and bulk HTSC. These models significantly reduce computational time compared to high level Finite Element Method (FEM), while still retaining system losses which lead to the non-linear, hysteretic force displacement behavior. The Translational Lumped Parameter model presented in this paper is an extension of the axisymmetric lumped parameter model presented in previous work.

**Model Development**

The HTSC puck is modeled as discrete, overlapping disc elements as shown in the figure below. Each superconducting disc element is associated with an uniform circulating current, $I$, that is induced per Faraday’s Law of Induction, with respect to movement of the permanent magnet. A power-law method is used to model the nonlinear voltage loss term, that represents system losses and saturates circulating currents to the critical current density of the bulk HTSC [2].

**Magnetic Coupling Representation**

The magnetic field generated by the permanent magnet is represented by discrete, constant value, filament current loops on the surface of the permanent magnet. An expansion of the Biot-Savart law can be used to calculate the magnetic vector potential, $A$, at any position with respect to the filament current loop [3]. Integrals of $A$ can be taken to calculate the magnetic flux, and flux gradients which penetrate each of the superconducting disc elements that make up the bulk HTSC model. These flux gradients can be viewed as gyrorot modulli, which represent the lossless magneto-mechanical interaction between induced currents within the bulk HTSC and resulting forces on the permanent magnet.

**Static Force Validation**

To verify performance of the transverse lumped parameter model, comparisons were made to experimental test data. A YBCO bulk superconductor (47mm OD x 15mm H) was field cooled using liquid nitrogen with an N48 neodymium magnet placed 2 mm above the top surface. The bulk YBCO superconducting was swept over a +/- 10 mm displacement over a 30s period, and translational and vertical reaction force measurements were made on the magnet. The lumped parameter model was setup to simulate the test conditions, with assumed material properties. Lump parameter models 1 and 2 assumed a critical current density of 9 kA/cm² and 10 kA/cm², respectively, and a magnetic coercive strength of 760 kA/m and 812 kA/m, respectively. The proposed lumped parameter formulation could be solved with a fixed step, 4th order Runge Kutta solver on a personal computer in a time span of 10 – 15 minutes. This solution time is a significant reduction over 3D FEM solvers, which can take over 48 hours to solve this dynamic problem.

**Dynamic Validation**

A second validation test was performed to evaluate dynamic performance of the proposed lumped parameter model. 6 YBCO superconductors (67mm x 35mm) were assembled to create a bottom bearing plate for an N42 neodymium magnet (76mm OD x 31mm ID x 25mm H). The YBCO bulks were field cooled with the magnet assembly placed 2mm above the surface. A 1 mm displacement step input was applied to the YBCO base and laser displacement sensors measured the response of the levitated magnet assembly. The step input creates a dynamic response which cannot be used for system identification, such as determined the linear stiffness. The lumped parameter model showed good performance in matching the step response test and local stiffness as shown in the time domain response and FFT plots below. Additional viscous damping was added to the lumped parameter model to account for friction between the magnet and liquid nitrogen in the cryostat.

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**Publications**

3. C.S. Hearn, Design Methodologies for Advanced Flywheel Energy Storage. Dissertation submitted to The University of Texas at Austin, August 2013

**References**